PARTICLE PHYSICS

Tutorials (n'3)

1. *Møller* **scattering.-** We consider a QED-like interaction structure for spinless particles S^{-1} (unique species of scalar field ϕ of mass m), charged by one unit: $Q = -1$, interacting, through the coupling constant $g = e$, with the vector boson field A^μ of the massless spin-one photon: $V \equiv \gamma$.

   (a) Draw all the possible *Feynman* diagrams contributing to the scattering reaction,

   $$S^{-1}(p_A^\nu)S^{-1}(p_B^\nu) \to S^{-1}(p_C^\nu)S^{-1}(p_D^\nu)\,,$$

   where the 4-momentum associated to each ingoing/outgoing particle is indicated. Note that if the particle S^{-1} is replaced by an electron e^- (spin 1/2), this reaction becomes the so-called *Møller* scattering. Indicate on each leg the arrow of propagation flux direction.

   (b) Write the main probability amplitude $-i\mathcal{M}_i$ for each of the processes drawn in previous question [1], and then main probability amplitude $-i\mathcal{M}$ for the global reaction, at leading order in the coupling constant e.

   (c) Express the whole amplitude obtained, $-i\mathcal{M}$, only in terms of e and the three *Mandelstam* variables:

   $$s \hat{=} (p_A^\nu + p_B^\nu)(p_{A\nu} + p_{B\nu}) \hat{=} (p_A^\nu + p_B^\nu)^{\underline{2}}\,,$$
   $$t \hat{=} (p_D^\nu - p_B^\nu)^{\underline{2}}\,,$$
   $$u \hat{=} (p_C^\nu - p_B^\nu)^{\underline{2}}\,.$$

   Use the feature that the external particles are on shell as well as the 4-momentum conservation relations.

   (d) Calculate now this amplitude $-i\mathcal{M}$ in the center-of-mass frame, where by definition

   $$\vec{p}_A + \vec{p}_B = \vec{0}\,,$$

   and within the case of relativistic scattered particles [2]. For this purpose, start by combine and apply all the informations and relevant properties on the four involved 4-momenta. Express $-i\mathcal{M}$ exclusively as a function of e and the θ angle between the direction of the initial particle with momentum p_A and the final p_C-particle beam. Comment physically the resulting formula.

[1] The table of *Feynman* rules drawn during the lectures can thus be used here.

[2] Negligible mass term with respect to the momentum one in the squared energy expression.

2. *Bhabha* **scattering.-** We consider a QED-like interaction structure for spinless particles S^{-1} (unique species of scalar field ϕ of mass m), charged by one unit: $Q = -1$, interacting, through the coupling constant $g = e$, with the vector boson field A^{μ} of the massless spin-one photon: $V \equiv \gamma$. The anti-particle is thus noted S^{+1}.

   (a) Draw all the possible *Feynman* diagrams [3] contributing to the scattering reaction,

   $$S^{-1}(p_A^{\nu})S^{+1}(p_B^{\nu}) \to S^{-1}(p_C^{\nu})S^{+1}(p_D^{\nu})\,,$$

   where the 4-momentum associated to each ingoing/outgoing particle is indicated. Note that if the particle S^{-1} is replaced by an electron e^{-} and S^{+1} by the positron e^{+} (spins 1/2), this reaction becomes the so-called *Bhabha* scattering. Indicate on each leg the arrow of propagation flux direction.

   (b) Write the main probability amplitude $-i\mathcal{M}_i$ for each of the processes drawn in previous question [4], and then main probability amplitude $-i\mathcal{M}$ for the global reaction, at the tree level.

   (c) Express the whole amplitude obtained, $-i\mathcal{M}$, only in terms of e and the three *Mandelstam* variables. Use the feature that the external particles are on shell as well as the 4-momentum conservation relations.

   (d) Calculate now this amplitude $-i\mathcal{M}$ in the center-of-mass frame and within the case of relativistic scattered particles. For this purpose, start by combine and apply all the informations and relevant properties on the four involved 4-momenta. Express $-i\mathcal{M}$ exclusively as a function of e and the θ angle between the direction of the initial particle with momentum p_A and the final p_C-particle beam. Comment physically the resulting formula.

3. **General kinematics relation.-** Demonstrate that the sum of the three *Mandelstam* variables is equal to the sum of the external particle squared masses, namely $s + t + u = 4m^2$, for any $2 \to 2$-body reaction of type,

   $$S^{Q}(p_A^{\nu})S^{Q}(p_B^{\nu}) \to S^{Q}(p_C^{\nu})S^{Q}(p_D^{\nu})\,,$$

   where the two ingoing and the two outgoing particles have a common mass m.

[3] Think of using the *Feynman* anti-particle prescription to recover the known *Feynman* rules.
[4] The table of *Feynman* rules drawn during the lectures can thus be used here.