Decoherence in quantum field theory

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[A. Giraud , J.S. : submitted to PRL]

- Motivations and set-up
- Nonequilibrium QFT: methods
- Results
- Conclusions and outlook

MOTIVATIONS

Measurement theory

Ultracold atoms

Quantum optics

Quantum decoherence

...

Quarks-gluons plasma physics

Neutrino physics

Early-Universe cosmology

Topological defects

Quantum-to-classical transition of primordial density fluctuations

Decay of inflaton (preheating)

...
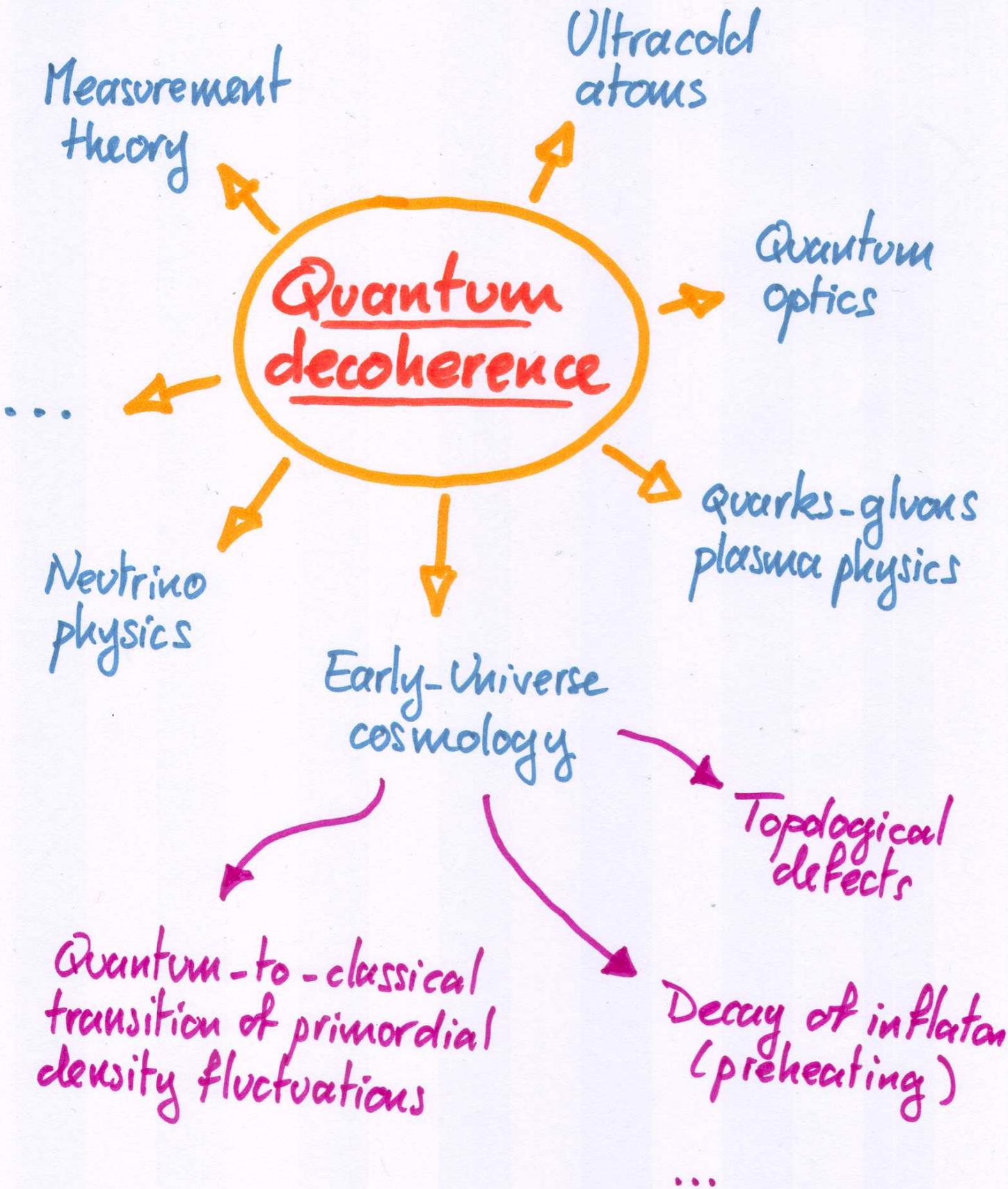
The standard picture

Decoherence : dynamical diagonalization of the density matrix (in a given basis)

ex:

$$\rho(t=0) = |\psi\rangle\langle\psi| \rightsquigarrow \rho(t) = \sum_n p_n |n\rangle\langle n|$$

pure state   statistical mixture

(open) quantum system ⟷ "environment" or "measurement apparatus" or "fast d.o.f." (large ⇒ classical)

Loss of purity / coherence results from the interaction with an "environment" or from some kind of coarse graining ⟹ NON-UNITARY EVOLUTION

Decoherence for a closed system

🟠 Reconstructing the actual state of the system requires one to measure all independent corre--lation functions.

🟠 For systems with large (infinite) number of d.o.f., this is practically impossible.

➡️ The inability to reconstruct the actual state of the system due to restricted data may result in apparent decoherence.

Information spreads in the space of corre--lation functions.
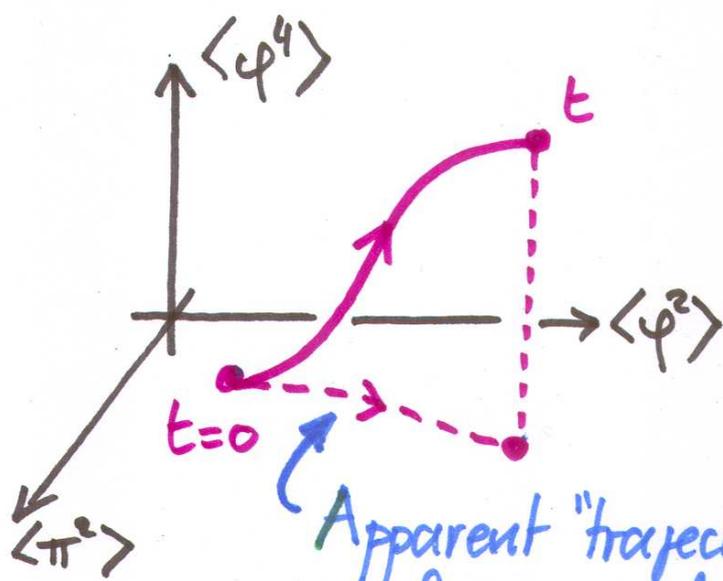
Apparent "trajectory" in the subspace of measured correlation functions

[e.g. Balian ('00) ; Campo, Parentani ('05) ...]

Theoretical description

Decoherence : a nonequilibrium quantum process.

- ■ **Analytical descriptions**
  - ● exactly solvable (simple) models
  - ● neglect the dynamics of the environment (e.g. assume a thermal bath)
  - ● neglect back-reaction of the system
  - ● weak coupling techniques

- ■ **Numerical work**

  - ● solve open quantum dynamics obtained by integrating out the environment d.o.f.

Real-time quantum dynamics

Nonequilibrium QFT : methods

⚠ **Standard approximation schemes in QFT fail out of equilibrium.**

- ⬤ **SECULARITY** : $e^{-\gamma t} \approx 1 - \gamma t + \ldots$

  ⇨ **Need for (infinite) resummations**

- ⬤ **UNIVERSALITY** : late-time thermalization (effective loss of memory)

  ⇨ **Need for non-linear approximation schemes**

Two-particle-irreducible (2PI) techniques
[e.g. Serreau @ SEWM 08 hep-ph/0410330]

$$\Gamma_{2PI}[\phi, G] = S[\phi] + \frac{i}{2} Tr \, Ln \, G^{-1} + \frac{i}{2} Tr \, G_0^{-1} G + \Gamma_2[\phi, G]$$

$$\left. \frac{\partial \Gamma_{2PI}}{\partial G} \right|_{\bar{G}} = 0 \quad \Leftrightarrow \quad \boxed{\bar{G}^{-1} = G_0^{-1} - \Sigma[\bar{G}]}$$

↖ Self-adjusting propagator

2PI techniques : State of art

➡ Solve the secularity and universality problems of nonequilibrium QFT. [Berges, Cox ('01)]

■ **Applications**

● Thermalization from first principles
  [Berges, Borsányi, J.S. ('03) ; Cooper, Dawson, Mihaila (bs)]

● Nonperturbative $1/N$ expansion
  [Aarts, Ahrensmeier, Baier, Berges, J.S. ('02)]

● Dynamics at nonperturbatively high densities
  e.g. preheating after inflation [Berges, J.S. ('03) Smit, Tranberg ('04)]

● Expanding geometries [Tranberg ('08)]
⋮

■ **Formal developments**

● 2PI renormalization theory
[Van Hees, Knoll ('02) ; Blaizot, Iancu, Reinosa ('03)
Berges, Borsányi, Reinosa, J.S ('05) ...]

● Gauge field theories [Reinosa, J.S. ('06)]
⋮

Decoherence in QFT : The model

- $O(N)$ scalar field φ_a, $a = 1 \dots N$

$$S[\varphi] = \int d^4x \left[\frac{1}{2} \partial_\mu \varphi_a \partial^\mu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right]$$

- 2PI $\quad 1/N$ - expansion at NLO [Berges ('02)]
[Aarts, Ahrensmeier, Baier, Berges, J.S. ('02)]

$$\Gamma_2[G] = \Gamma_2^{LO}[G] + \Gamma_2^{NLO}[G] + \dots$$

$$\infty \sim \frac{\lambda}{N} N^2 \sim \lambda N$$

$$\sum_P \underset{3}{\overset{1 \quad P}{2}} \sim \frac{\lambda^P}{N^P} N^P \sim N^0$$

Gaussian dynamics : no "decoherence"
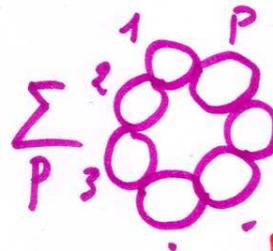
Non-Gaussian

$$\Sigma[G] = 2i \frac{\delta \Gamma_2}{\delta G} = \underset{LO}{\underbrace{\bigcirc}} + \underset{\text{NLO (scattering memory effects, ...)}}{\underbrace{\ominus + \dots + \bowtie}} + \dots$$

LO (mass correction)

Equations of motion

$$G(x,y) = \langle T\varphi(x)\varphi(y)\rangle = F(x,y) - \frac{i}{2}\varepsilon(x^0-y^0)\rho(x,y)$$

$$[\Box + M^2(x)]\,F(x,y) = -\int_0^{x^0} dz\,\Sigma_\rho(x,z)F(z,y) + \int_0^{y^0} dz\,\Sigma_F(x,z)\rho(z,y)$$

$$[\Box + M^2(x)]\,\rho(x,y) = -\int_{y^0}^{x^0} dz\,\Sigma_\rho(x,z)\rho(z,y)$$

(local) mass corrections

Non local terms
= memory / scatterings

$$\Sigma_F(x,y) = \frac{\lambda}{3N}\left[F(x,y)\,I_F(x,y) - \frac{1}{4}\rho(x,y)\,I_\rho(x,y)\right]$$

$$\Sigma_\rho(x,y) = \frac{\lambda}{3N}\left[F(x,y)\,I_\rho(x,y) + \rho(x,y)\,I_F(x,y)\right]$$

$$I_F(x,y) = \Pi_F(x,y) - \int_0^{x^0} dz\,I_\rho(x,z)\Pi_F(z,y) + \int_0^{y^0} dz\,I_F(x,z)\Pi_\rho(z,y)$$

$$I_\rho(x,y) = \Pi_\rho(x,y) - \int_{y^0}^{x^0} dz\,I_\rho(x,z)\Pi_\rho(z,y)$$

$$\Pi_F(x,y) = -\frac{\lambda}{6}\left[F^2(x,y) - \frac{1}{4}\rho^2(x,y)\right]\ ;\quad \Pi_\rho(x,y) = -\frac{\lambda}{3}F(x,y)\rho(x,y)$$

The picture

Assume one only measures equal-time 2-point functions:

$$F_p(t) = \langle \varphi_p^\dagger(t) \varphi_p(t) \rangle \quad ; \quad K_p(t) = \langle \pi_p^\dagger(t) \pi_p(t) \rangle$$

$$R_p(t) = \frac{1}{2} \langle \pi_p^\dagger(t) \varphi_p(t) + \varphi_p^\dagger(t) \pi_p(t) \rangle$$
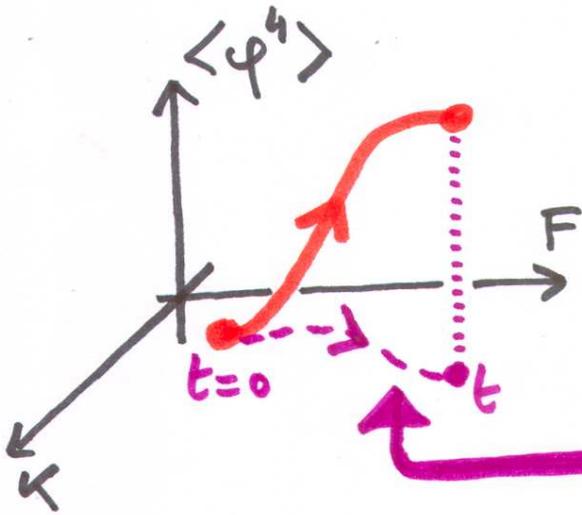
The least-biased (effective) density matrix compatible with the known data:

$$\rho_{eff}(t) = \prod_p \rho_p(t)$$

$$\rho_p(t) \propto \exp\left[k_p(t) \left(\bar{F}_p(t) \pi_p^\dagger \pi_p + K_p(t) \varphi_p^\dagger \varphi_p + R_p(t) (\varphi_p^\dagger \pi_p + \pi_p^\dagger \varphi_p) \right) \right]$$

$$k_p(t) = - \frac{\ln\left(1 + 1/n_p(t) \right)}{2 n_p(t) + 1} \quad ; \quad n_p(t) + \frac{1}{2} = \sqrt{F_p(t) K_p(t) - R_p^2(t)}$$

N.B.: $$\boxed{ \operatorname{tr} \rho_p^2(t) = \frac{1}{2 n_p(t) + 1} }$$: a measure of **purity**

The Gaussian density matrix

Parametrization:

$0 \leq \phi < 2\pi$

$0 \leq \gamma < 1$

$$\varepsilon F = \left(n + \frac{1}{2}\right) \frac{1 - \gamma \cos\phi}{\sqrt{1 - \gamma^2}}$$

$$R = -\left(n + \frac{1}{2}\right) \gamma \frac{\sin\phi}{\sqrt{1 - \gamma^2}}$$

$$K/\varepsilon = \left(n + \frac{1}{2}\right) \frac{1 + \gamma \cos\phi}{\sqrt{1 - \gamma^2}}$$

$\boxed{\varepsilon}$ = energy scale : fixes the basis
we take $\varepsilon_p(t) = \sqrt{p^2 + M^2(t)}$

$\boxed{\rho_p(t)}$

$\Pi/\sqrt{\varepsilon}$

$\phi/2$

$\varphi\sqrt{\varepsilon}$

$$\sqrt{\left(n + \frac{1}{2}\right)}\sqrt{\frac{1 + \gamma}{1 - \gamma}}$$

$$\sqrt{\left(n + \frac{1}{2}\right)}\sqrt{\frac{1 - \gamma}{1 + \gamma}}$$
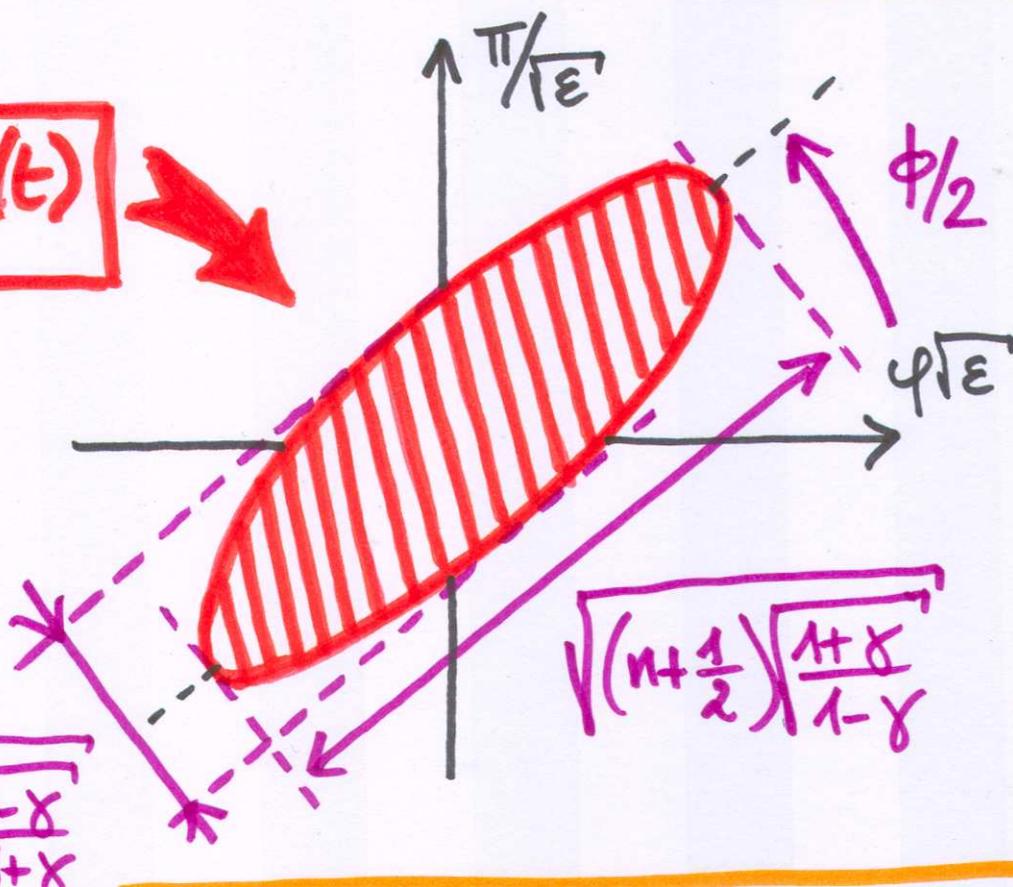
Observables

- $\text{tr}\,\rho_p^2(t) = \dfrac{1}{2n_p(t)+1} \leq 1$

  PURITY (basis-indep.)

  Pure state $(n=0)$

- $S_p(t) = -\text{tr}\,\rho_p(t)\ln\rho_p(t)$

 $= (n_p(t)+1)\ln(n_p(t)+1) - n_p(t)\ln n_p(t)$

  ENTROPY = MISSING INFORMATION (basis-indep.)

- $\gamma_p(t)$: squeezing parameter

  COHERENCE in two-modes coherent states basis [Campo, Parentani ('05)]

Initial conditions

We assume a __Gaussian initial state__

$$|\varphi'\rangle = \frac{|\varphi\rangle + |\pi\rangle}{\sqrt{2}}$$

$$\rho = |\varphi'\rangle\langle\varphi'|$$

$$\rho = |0\rangle\langle 0|$$

$n_0 = 0$
$\gamma_0 = 1$

$n_0 = 0,\ \gamma_0 = 0$

"condensate"   "vacuum"
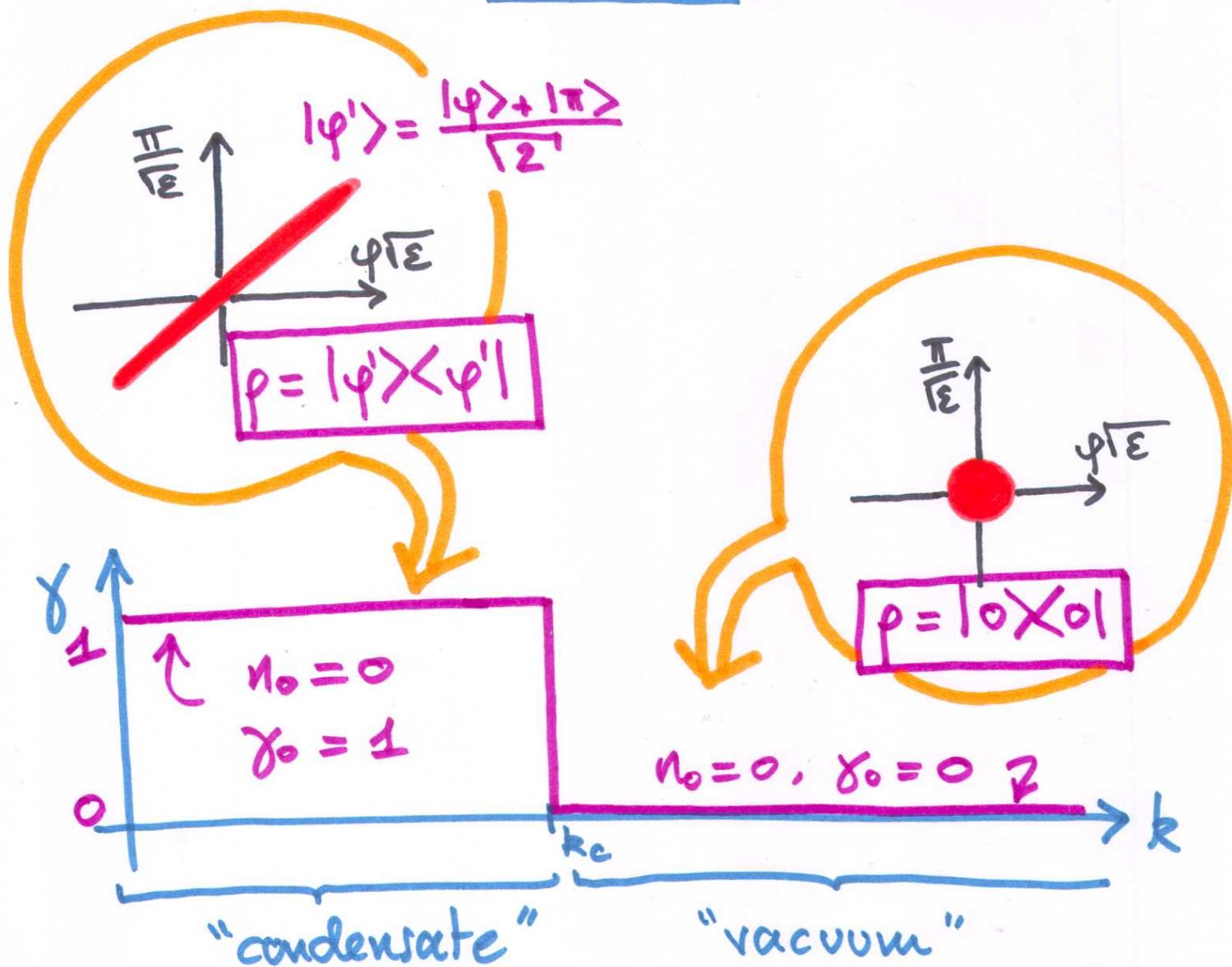
N.B. : No environment, no (thermal) bath...

Loss of quantum purity / coherence is to be caused by __vacuum__ __quantum fluctuations__ !!

Results

$\text{tr}\,\rho_p^2(t)$
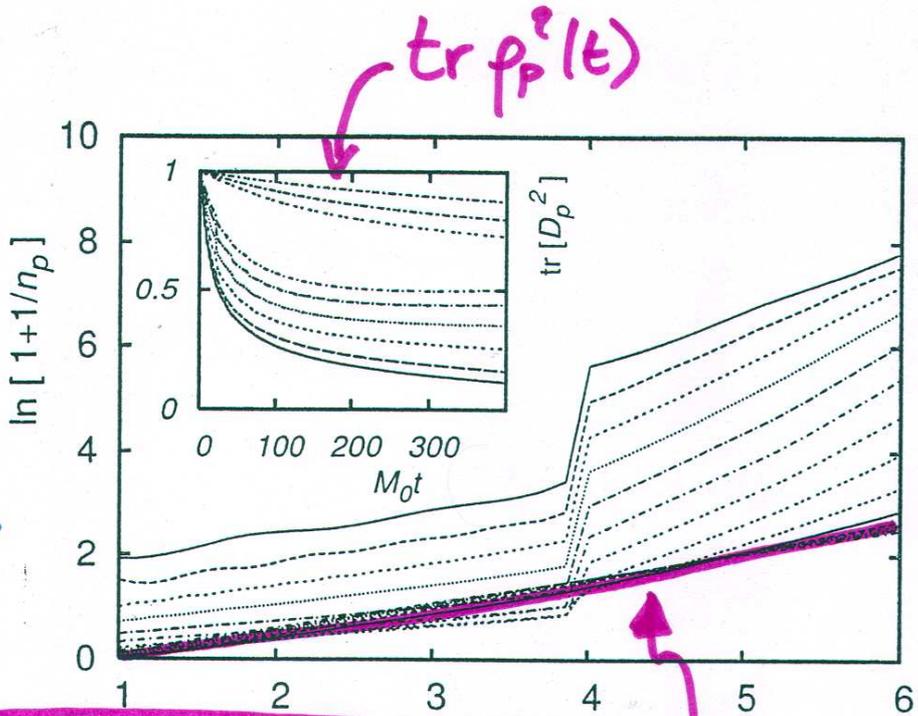
Apparent loss of purity and thermalization

Bose-Einstein distribution !

The pure state looks mixed and even (later) thermal to the observer who has restricted information .
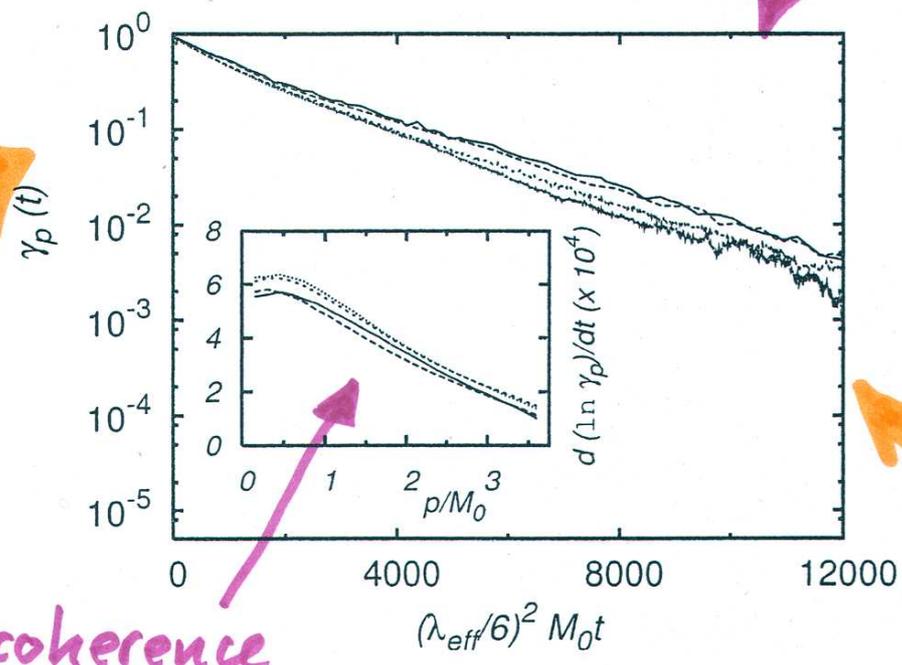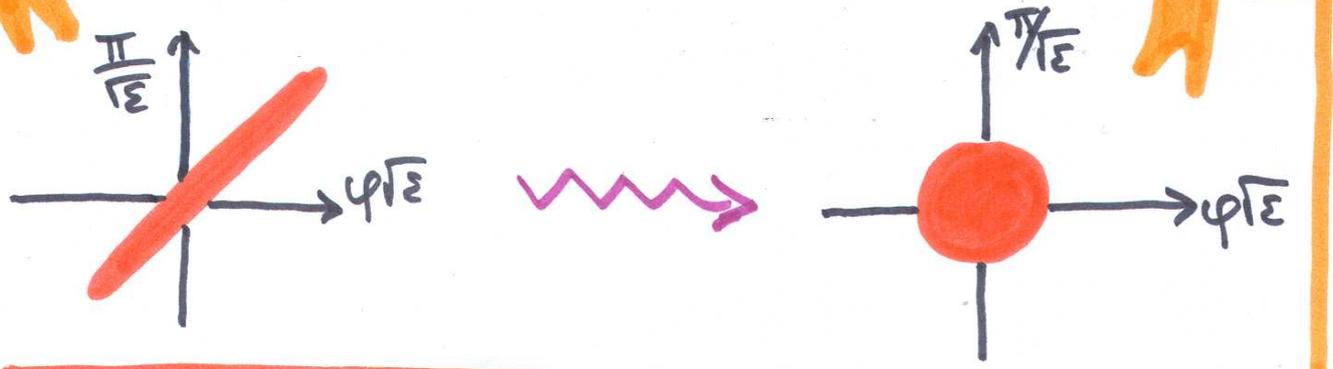
Classical scaling

$$F, R, K \sim \frac{1}{\sqrt{1-\gamma^2}} \underset{\gamma \to 1}{\gg} \rho \sim 1$$

Classical (statistical) fluctuations regime

$$S[\varphi] = \int d^4x \left(\frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!N}\varphi^4 \right)$$

$$= \eta \int d^4x \left(\frac{1}{2}(\partial\varphi')^2 - \frac{m^2}{2}\varphi'^2 - \frac{\lambda\eta}{4!N}\varphi'^4 \right)$$

$$\boxed{\varphi = \sqrt{\eta}\,\varphi'}$$

effective coupling

In the classical (strong) field regime, the results should only depend on

$$\lambda_{eff} = \frac{\lambda}{\sqrt{1-\gamma_0^2}}$$

Loss of purity/coherence can be described by means of classical statistical field theory

Conclusions ...

- effective loss of quantum purity / coherence due to restricted knowledge on the system.
- induced by pure <u>quantum fluctuations</u>.
- can be described by classical statistical (stochastic) field theory.

... and Outlook

➡ finite temperature (in progress)

➡ phase transition

➡ expanding geometries

➡ fermionic d.o.f.

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