Topics in $U(3)$ χPT dynamics and related spectroscopy.

J. A. Oller

Departamento de Física
Universidad de Murcia

in collaboration with Zhi-Hui Guo

Phys. Rev. D**84**(2011)034005, others forthcoming

Outline

1. Introduction
2. Analytical Calculation
3. Numerical results
4. SS, PP correlators
5. Average duality
6. N_C pole trajectories
7. Conclusions

Introduction

Along the years we have been very interested in the scalar dynamics and spectrum combining ChPT with non-perturbative techniques from S-matrix theory:

Unitarization techniques based on the N/D method, Bethe-Salpeter equation, Omnés functions, solving the Muskhelishvili-Omnés equation in coupled channels, etc.

Oset,JAO NPA620'97;NPA629'98;PRD'99

JAO, PLB'98;PLB'00;NPA'03;NPA'03;PRD'05

Oset, Peláez, JAO PRL'98; PRD'99

Meißner,JAO NPA'01

Jamin,Pich,JAO NPB'00;NPB'02;EPJC'02;JHEP'04;PRD'06

Roca,JAO EPJA'07;PLB'07;EPJA'08; **Roca,Schat**,JAO PLB'08

Albaladejo,JAO PRL'08; **Albaladejo,Roca**,JAO PRD'10

Guo,JAO PRD'11

Thanks! to all my collaborators

Now we want to study simultaneously important QCD results:

i) Large N_C QCD. Pole trajectories with varying N_C
$\bar{q}q$ resonances: $M \sim \mathcal{O}(N_C^0)$, $\Gamma \sim \mathcal{O}(1/N_c)$
Glueball resonances: $M \sim \mathcal{O}(N_C^0)$, $\Gamma \sim \mathcal{O}(1/N_c^2)$
Tetraquarks dissolve in the continuum
Oller, Oset PRD'99 $M_S^2 \propto f^2 \propto N_c$
Peláez *et al* PRL'04,PRL'06,...,PRD'11

ii) Second (Weinberg) spectral function sum rules for $SS - SS$ and $SS - PP$ correlators
Bijnens, Gámiz, Prades JHEP'01
Sanz-Cillero, Trnka PRD'10

iii) Average or semi-local duality
Peláez, Pennington, Ruiz de Elvira, Wilson PRD'11

The large N_C dependence of ii) and iii) are of great interest:

In the physical world the σ or $f_0(600)$ dominates the isoscalar scalar channel at low energies
The $f_0(600)$ disappears with growing N_C (e.g. for a tetraquark, a meson-meson molecule or resonance)

iii) (and also ii)) requires of a strong scalar contribution to achieve the cancellations needed for N_C large.

Which is its origin?

Interesting puzzle

Previous studies of the N_C trajectory for the $f_0(600)$ pole were based on $SU(2)$ or $SU(3)$ ChPT

However, in the large N_C limit:

- Neither $SU(2)$ nor $SU(3)$ ChPT have the right degrees of freedom
- The η_1 becomes the ninth Goldstone boson in the chiral limit
- The η becomes similarly light as the π for $N_C \to \infty$
  Weinberg PRD'75

In the chiral limit $m_u = m_d = m_s = 0$ the QCD Lagrangian is invariant under $U_L(3) \otimes U_R(3)$ symmetry.

$SU_L(3) \otimes SU_R(3) \to SU_V(3)$ is Spontaneously Broken. Goldstone bosons appear π, K, η

$U_V(1) \equiv U_{L+R}$ Conserved Baryon Number.

$U_A(1) \equiv U_{L-R}$ Neither Conserved nor Goldstone Boson.

Puzzle:

Goldstone mode: There should be an η_1 with a mass $< \sqrt{3} m_\pi$ Weinberg PRD'75 but η is much heavier.

The ninth axial singlet current has an anomalous divergence Adler PR'69 Fujikawa PRD'80

$$J_5^{\mu\,(0)} = \bar{q}\gamma_\mu\gamma_5 q$$
$$\partial_\mu J_5^{\mu\,(0)} = \frac{g^2}{16\pi^2}\frac{1}{N_c} Tr_c(G_{\mu\nu}\widetilde{G}^{\mu\nu})$$

Large N_c QCD $N_c \to \infty$, $g^2 N_c \to$ *constant*
't Hooft NPB'74, Witten NPB'79

$U_L(N_F) \otimes U_R(N_F) \to U_{L+R}(N_F)$
Entire Nonet of Goldstone bosons results.

Coleman, Witten PRL'80
Knecht, de Rafael PLB'98

The explicit breaking of chiral symmetry due to quark masses and the $U_A(1)$ anomaly is treated perturbatively.

Combined power expansion in the light quark masses and $1/N_c$

This formalism is set up in
Di Vecchia, Veneziano NPB'80
Rosenzweig, Schechter, Trahern PRD'80
Witten Ann.Phys.'80
The Leading Order in $1/N_c$ and the Derivative Expansion was worked out.

Herrera-Siklody, Latorre, Pascual, Taron NPB'97
Generalization of Gasser, Leutwyler Ann.Phys.'84, NPB'85 from $SU_L(3) \otimes SU_R(3)$ to $U_L(3) \otimes U_R(3)$

Generating functional in the presence of external sources $\mathcal{Z}[l, r, s, p, \theta]$. Chiral Lagrangian to $\mathcal{O}(p^4)$ and all orders in $1/N_c$

Bilinear Quark operators (currents) and the Topological Charge operator coupled to external sources:

$$\mathcal{L} = \mathcal{L}_{QCD} + \bar{q}_L \gamma_\mu \ell^\mu(x) q_L + \bar{q}_R \gamma_\mu r^\mu(x) q_R - \bar{q}_R (s(x) + ip(x)) q_L$$
$$- \bar{q}_L (s(x) - ip(x)) q_R - \frac{g^2}{16\pi^2} \frac{\theta(x)}{N_c} Tr_c(G_{\mu\nu} \widetilde{G}^{\mu\nu})$$

$$g_L = I + i(\beta - \alpha) \; , \; g_R = I + i(\beta + \alpha)$$
$$\theta(x) \to \theta(x) - 2\langle \alpha(x) \rangle$$

The extra term in the fermionic determinant due to the anomaly is compensated.

The non-Abelian anomaly cannot be compensated. The Wess Zumino Witten term has to be added by hand
Wess and Zumino PLB'71 , Witten NPB'83
Adler, PR'69, Bardeen, PR'69 , Adler and Bardeen, PR'69

$$D_\mu U \to g_R (D_\mu U) g_L^\dagger$$

$$U = \exp\left(i \frac{\sqrt{2}}{f} \Phi \right) ,$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}$$

This reflects the pseudo-Goldstone nature of the η_1

$N_c \to \infty$ enforces a nonet symmetry, with the π, K, η and η' all having identical properties.

There is then only one decay constant f for the octet and singlet of pseudo-Goldstone bosons.

In addition one has the specific gluonic content of the η_1 due to $U_A(1)$ anomaly, which explicitly breaks chiral symmetry.

Both (pseudo-Goldstone boson+ gluonic content) are related by the combination

$$X(x) = \langle \log U(x) \rangle + i\theta(x) \equiv i\frac{\sqrt{2N_F}}{f}\eta_1 + \hat{\theta}(x)$$

is invariant as well as any function of X Witten NPB'79, Leutwyler PLB'96

$\theta(x)$ couples in the QCD Lagrangian through the topological charge

$$-\frac{g^2}{16\pi^2}\frac{Tr\left(G_{\mu\nu}\widetilde{G}^{\mu\nu}\right)}{N_c} \ .$$

Extra suppression factor $1/N_c$.

Vertex suppression $1/N_c^{3/2}$ per η_1

Leading Order Lagrangian in the Chiral Expansion:

$$\mathcal{L}_{0+2} = -W_0(X) + W_1(X)\langle D_\mu U^\dagger D^\mu U\rangle + W_2(X)\langle U^\dagger \chi + \chi^\dagger U\rangle$$
$$+ iW_3(X)\langle U^\dagger \chi - \chi^\dagger U\rangle + W_4(X)\langle U^\dagger D_\mu U\rangle\langle U^\dagger D^\mu U\rangle$$
$$+ W_5(X)\langle U^\dagger (D_\mu U)\rangle D^\mu \hat{\theta} + W_6(X) D_\mu \hat{\theta} D^\mu \hat{\theta}$$

$W_4(0) = 0$, $W_1(0) = W_2(0) = \frac{f^2}{4}$ from quadratic fields

$$M^2_{\eta_1}\Big|_{U_A(1)} = -\frac{2N_F}{f^2} W_0''(0) \propto \frac{1}{N_c}$$

Herrera-Siklody *et al.* NPB'97

$$\mathcal{L}_4 = \sum_{i=0}^{57} c_i(\mu) O_i$$

The β_i functions are also calculated.

These Lagrangians were employed by Borasoy *et al*

- S-wave meson-meson scattering from unitarized $U(3)$ chiral Lagrangians PRD'03
  The interaction kernel is calculated only at the tree-level
- Photoproduction off nucleons of η and η' PRD'01
- One-loop calculation in Infrared Regularization for:
  η-η' mixing EPJA'01
  $\eta' \to \eta\pi\pi$ decay NPA'02

It is stressed that $M_{\eta'}$ is large.

But $M_{\eta'}$ also appears from vertices and there is nothing like 'Baryon number conservation' that acts in the meson-baryon sector.

Proliferation of free parameters. Poor predictive power.

$1/N_c$ counting for counterterms Leutwyler PLB'96

$$G(X) = g(\frac{X}{N_c})\, N_c^{2-N(Tr_F)-N(\hat{\theta})}$$

for Arbitrary Number of Flavors.

The expansion of g in powers of X/N_c has only leading coefficients of order 1.

$f \sim \sqrt{N_c} \rightarrow$ Each additional Meson Loop is suppressed by N_c^{-1}

δ-expansion: $p^2 \sim m_q \sim 1/N_c \sim \delta$

Loops are suppressed by $p^2/f^2 \sim \mathcal{O}(\delta^2)$

For low energy implications of $U(3)$ theory Leutwyler PLB'96; Kaiser and Leutwyler EPJC'00

Lagrangians up to NLO, $\mathcal{O}(\delta)$, have been thoroughly studied
Herrera-Siklody *et al.* NPB'97; Kaiser, Leutwyler, EPJC'00

$\mathcal{L}_{\delta^0}$: B, F, M_0^2

$\mathcal{L}_{\delta}$: 11 operators.

- $\eta-\eta'$ mixing:
  NLO $\mathcal{O}(\delta^2)$ Herrera-Siklody *et al.* PLB'98
  One loop calculation, plus the pseudoscalar decay constants
  Kaiser, Leutwyler hep-ph/9806336

The η-η' mixing does not follow the naive one-mixing-angle scheme.

- $\eta' \to \eta\pi\pi$: Tree level input at $\mathcal{O}(\delta^2)$ with $\pi\pi$ Final State Interactions resummed employing Unitary χPT based on the N/D method Escribano, Masjuan, Sanz-Cillero, JHEP'11
  However, they take the mass and mixing angle for η, η' from other works. Inconsistency between input and output.

Spectral function Sum Rules for $SS-SS$ and $SS-PP$ correlators

$$S_s^{(a)}(s)=\bar{q}(x)\frac{\lambda^a}{\sqrt{2}}q(x)\ ,\ \ P^{(a)}(x)=\bar{q}(x)i\gamma_5\frac{\lambda^a}{\sqrt{2}}q(x)$$

$$\Pi_{S,P}(Q)=-i\int d^4x\, e^{iqx}\, T\langle S,P(x)S,P(0)\rangle$$

Constraint: Non-trivial relation between the scalar spectrum (resonances) and the pseudoscalar one (Goldstone bosons and resonances) in the chiral limit.

Bernard, Duncan, LoSecco, Weinberg PRD'75

$$\int_0^\infty ds\left[\text{Im}\Pi_S^{(0)}(s)-\text{Im}\Pi_P^{(3)}(s)\right]=0=\int_0^\infty ds\left[\text{Im}\Pi_S^{(3)}(s)-\text{Im}\Pi_P^{(0)}(s)\right]$$

Moussallam EPJC'99, HEP'00; Leutwyler, NPB'90

$$\int_0^\infty \text{Im}\Pi_{SS}^{(0-3)}(s)ds=0=\int_0^\infty \text{Im}\Pi_{PP}^{(0-3)}(s)ds$$

• We perform the first complete calculation of the meson-meson scattering within $U(3)$ χPT at one loop level, with monomials of $\mathcal{O}(\delta^2)$ and $\mathcal{O}(\delta^3)$.

• We include explicit exchange of tree level resonances instead of local counterterms: V (1^{--}), S (0^{++}) and P (0^{-+}) resonances.

• These amplitudes are then unitarized so that we can study resonances and their properties.

- Are the results stable under the inclusion of the η_1?
- η_1 becomes in large N_c the ninth pseudo-Goldstone boson. $SU(3)$ χPT results are ill-defined for $N_c \to \infty$.
- In large N_c there should be a light isosinglet pseudoscalar with mass $\sim m_\pi$
- Influence on the running of the pole positions with Large N_c.

Jamin,Oller,Pich, NPB'00 I$=1/2$ S-wave meson-meson scattering
It was not a full one-loop calculation for the kernel.

Beisert and Borasoy, PRD'03 studied S-wave meson-meson scattering from $\mathcal{L}_{\delta^0}$ and $\mathcal{L}_\delta$
The interaction kernel is calculated at tree level.
Local terms instead of resonance exchanges.

Another framework is non-relativistic effective field theory Kubis and Schneider EPJC'09 studied cusp effects in $\eta' \to \eta\pi\pi$ similarly to $K \to 3\pi$ Colangelo,Gasser,Kubis,Rusetksy PLB'06

Relevant Chiral Lagrangian

$$\mathcal{L}^{(0)} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{3} M_0^2 \left(\ln \det u\right)^2$$

where

$$u = e^{i\frac{\Phi}{\sqrt{2}F}} \;,\; U = u^2 \;,$$
$$u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger \;,\; \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$\Phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0 + \eta_8 + \sqrt{2}\eta_1}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0 + \eta_8 + \sqrt{2}\eta_1}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8 + \sqrt{2}\eta_1}{\sqrt{6}} \end{pmatrix}$$

L_i's generically correspond to the higher order local operators.

At $\mathcal{O}(\delta)$ one has $\mathcal{O}(N_c p^4)$ and $\mathcal{O}(p^2)$ operators:

$$\begin{aligned}\mathcal{L}^{(\delta)} &= L_2\langle u_\mu u_\nu u^\mu u^\nu\rangle + (2L_2+L_3)\langle u_\mu u^\mu u_\nu u^\nu\rangle \\ &+ L_5\langle D_\mu U^\dagger D^\mu U(U^\dagger\chi+\chi U)\rangle + L_8\langle U^\dagger\chi U^\dagger\chi+\chi^\dagger U\chi^\dagger U\rangle + \ldots \\ &+ F^2\Lambda_1\langle u_\mu\rangle\langle u^\mu\rangle + F^2\Lambda_2 \ln(\det U)\langle U^\dagger\chi-\chi^\dagger U\rangle + \ldots\end{aligned}$$

- Λ_2 is relevant for η-η' mixing. (Not for meson-meson scattering).
- Λ_1 only gives rise to η_1 field renormalization. If included in the fit its resulting value vanishes.

$\mathcal{O}(p^4)$

$$\mathcal{L}^{(\delta^2)} = (L_1 - L_2/2)\langle u_\mu u^\nu \rangle^2 + L_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle + \ldots$$
$$+ L_{18} i D_\mu X \langle D^\mu U^\dagger \chi - D^\mu U \chi^\dagger \rangle + L_{25} i X \langle U^\dagger \chi U^\dagger \chi - \chi^\dagger U \chi^\dagger U \rangle + \ldots$$

[Kaiser and Leutwyler, '00] [Herrera-Siklody, *et al.*, '97]

- In the current discussion, we assume the resonance saturation of ChPT counterterms and exploit resonance exchanges to calculate the meson-meson scattering.
- The monomials proportional to Λ_1 and Λ_2 are not generated through resonance exchange. No double counting.

Vector, Scalar and Pseudoscalar resonance operators:

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$\begin{aligned}\mathcal{L}_S =\ & c_d\langle\, S_8 u_\mu u^\mu \,\rangle + c_m\langle\, S_8\chi_+ \,\rangle \\ & + \widetilde{c}_d S_1\langle\, u_\mu u^\mu \,\rangle + \widetilde{c}_m S_1\langle\, \chi_+ \,\rangle + \hat{c}_d\langle\, S_9 u_\mu \,\rangle\langle\, u_\mu \,\rangle\end{aligned}$$

$$\mathcal{L}_P = \ i d_m\langle P_8\chi_-\rangle + i\widetilde{d}_m P_1\langle\chi_-\rangle$$

Ecker, Gasser, Pich, de Rafael NPB'89

Elimination of the mixing terms between P resonances and pseudo-Goldstone bosons:

$$\begin{aligned} P_8 &\to \overline{P}_8 + i\frac{d_m}{M^2_{P_8}}\left(\chi_- - \frac{1}{3}\langle\, \chi_- \,\rangle I_{3\times 3}\right), \\ P_1 &\to \overline{P}_1 + i\frac{\widetilde{d}_m}{M^2_{P_1}}\langle\, \chi_- \,\rangle\,, \end{aligned}$$

$$\mathcal{L}_{\overline{P}} = \frac{1}{2}\langle\, \nabla^\mu \overline{P}_8 \nabla_\mu \overline{P}_8 - M^2_{\overline{P}_8} \overline{P}_8^2 \,\rangle + \frac{1}{2}\big(\partial^\mu \overline{P}_1 \partial_\mu \overline{P}_1 - M^2_{\overline{P}_1} \overline{P}_1^2\big)$$
$$+ i\frac{d_m}{M^2_{\overline{P}_8}}\langle\, \nabla_\mu \overline{P}_8\, \nabla^\mu \chi_- \,\rangle + i\frac{\widetilde{d}_m}{M^2_{\overline{P}_1}} \nabla_\mu \overline{P}_1\, \nabla^\mu \langle\, \chi_- \,\rangle$$
$$- \frac{d_m^2}{2M^2_{\overline{P}_8}}\langle\, \chi_- \chi_- \,\rangle + \big(\frac{d_m^2}{6M^2_{\overline{P}_8}} - \frac{\widetilde{d}_m^2}{2M^2_{\overline{P}_1}}\big)\langle\, \chi_- \,\rangle\langle\, \chi_- \,\rangle + \cdots \,,$$

However, the properties of the pseudoscalar resonances are not well settled:

$$\frac{\delta L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_- \rangle$$

No δL_7 because we assume the large N_C relations

$$\widetilde{d}_m = \frac{1}{\sqrt{3}} d_m \ , \ \ M_{P_1} = M_{P_8}$$

$$d_m = 30 \text{ MeV} \quad , \quad M_{P_8} = 1350 \text{ MeV}$$

Ecker *et al.* NPB'89; Golterman, Peris PRD'00; Sanz-Cillero, Trnka PRD'10

For the scalar resonances we always assume the large N_C constraint:

$$\widetilde{c}_d = \frac{1}{\sqrt{3}} c_d \quad , \quad \widetilde{c}_m = \frac{1}{\sqrt{3}} c_m$$

We include two scalar nonets

The second one with a bare mass $\gtrsim 2$ GeV is taken from Jamin, Pich, JAO NPB'00 since they were more sensitive to the more massive second scalar nonet:

$$c'_d = c'_m = \sqrt{3}\,\widetilde{c}'_d = \sqrt{3}\,\widetilde{c}'_m \simeq 40 \text{ MeV}$$

$$M_{S'_8} = M_{S'_1} \simeq 2.5\text{MeV}$$

Perturbative calculation of the scattering amplitudes

Figure: Relevant Feynman diagrams for the pseudoscalar self-energy.

The leading order η-η' mixing has to be solved exactly

$$\begin{aligned} \bar{\eta} &= \cos\theta\,\eta_8 - \sin\theta\,\eta_1\,, \\ \bar{\eta}' &= \sin\theta\,\eta_8 + \cos\theta\,\eta_1\,, \end{aligned}$$

$$\sin\theta = -1/\sqrt{1+\left(3M_0^2-2\Delta^2+\sqrt{9M_0^4-12M_0^2\Delta^2+36\Delta^4}\right)^2/32\Delta^4}$$

$$\Delta^2 = \overline{m}_K^2 - \overline{m}_\pi^2 \quad , \quad \sin\theta \to 0 \text{ for } \Delta^2 \to 0 \text{ or } M_0^2 \to \infty.$$

The NLO $\bar\eta$-$\bar\eta'$ mixing can be treated perturbatively

$$\begin{aligned}\mathcal{L} &= \frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}+\frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}'+\delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' \\ &\quad -\frac{m^2_{\overline{\eta}}+\delta_{m^2_{\overline{\eta}}}}{2}\overline{\eta}\,\overline{\eta}-\frac{m^2_{\overline{\eta}'}+\delta_{m^2_{\overline{\eta}'}}}{2}\overline{\eta}'\overline{\eta}'-\delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.\end{aligned}$$

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\theta_\delta & -\sin\theta_\delta\\ \sin\theta_\delta & \cos\theta_\delta\end{pmatrix}\begin{pmatrix}1+\frac{\delta_{\overline{\eta}}}{2} & \frac{\delta_k}{2}\\ \frac{\delta_k}{2} & 1+\frac{\delta_{\overline{\eta}'}}{2}\end{pmatrix}\begin{pmatrix}\overline{\eta}\\ \overline{\eta}'\end{pmatrix}.$$

Because of δ_k the one-mixing angle scheme for η-η' is not appropriate.

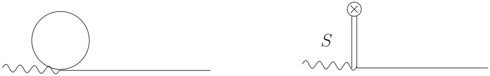


Figure: Relevant Feynman diagrams for the pseudoscalar decay constants.

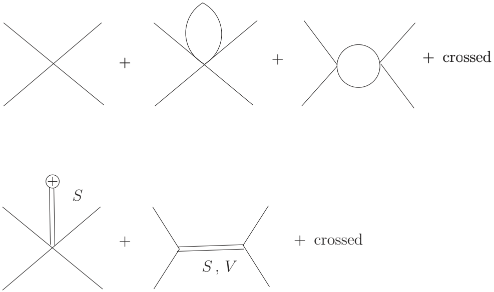
We expressed all the amplitudes in terms of physical masses and $F_\pi = 92.4$ MeV

Reshuffling of the LO contributions.

$$F_\pi = F\left\{1 + \frac{1}{16\pi^2 F_\pi^2}\left[A_0(m_\pi^2) + \frac{1}{2}A_0(m_K^2)\right] + \left[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2 + 2m_K^2)}{F_\pi^2 M_{S_1}^2} - \frac{8c_d\,c_m\,(m_K^2 - m_\pi^2)}{3F_\pi^2 M_{S_8}^2}\right]\right\}$$

This allows to know the N_C-dependence of F_π at the one-loop level in $U(3)$ ChPT including $1/N_C$ subleading terms

NLO Meson-meson scattering $\mathcal{O}(\delta^3)$:



We keep all contributions up to $\mathcal{O}(\delta^3)$ and calculate all the one-loop diagrams $\mathcal{O}(\delta^4)$

In addition, resonance exchanges generate an infinite tower of higher order contributions from resonance propagators

$$\frac{1}{M^2 - t} = \frac{1}{M^2}\left(1 + \frac{t}{M^2} + \frac{t^2}{M^4} + \ldots\right)$$

Not depicted

- Wave Function Renormalization
- η-η' Mixing:

  For NLO scattering only the LO mixing is required

  For LO scattering one needs the mixing up to NLO
- Rewriting of F in terms of F_π for the LO scattering

$$F_\pi = F\left\{1+\frac{1}{16\pi^2F_\pi^2}\left[A_0(m_\pi^2)+\frac{1}{2}A_0(m_K^2)\right]\right. \\ \left. +\left[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2+2m_K^2)}{F_\pi^2M_{S_1}^2}-\frac{8c_d\,c_m\,(m_K^2-m_\pi^2)}{3F_\pi^2M_{S_8}^2}\right]\right\}.$$

http://www.um.es/oller/u3FullAmp16.nb

Partial wave amplitude and its unitarization

Why unitarizing?

1. In $U(3)$ ChPT one has the large s quark mass and anomaly mass term.
   In many kinematical regions the two-body pseudoscalar thresholds are much larger than the typical three-momenta.
   This enhances the unitarity two-pseudoscalar reducible loop contributions Weinberg NPB91
2. We also go over the resonance region where the unitarity upper bound in partial waves is easily reached

Unitarity must be treated in a non-perturbative way

Partial wave projection:

$$T_J^I(s) = \frac{1}{2(\sqrt{2})^N} \int_{-1}^{1} dx\, P_J(x)\, T^I[s, t(x), u(x)]\,,$$

where $P_J(x)$ denote the Legendre polynomials and $(\sqrt{2})^N$ is a symmetry factor to account for the identical particles, such as $\pi\pi, \eta\eta, \eta'\eta'$.

This defines the Unitary Normalization Oller,Oset NPA'97

$$\mathrm{Im}\, T^I_{J\,mn} = \sum_k \theta(s - s^k_{th}) \rho_k\, T^I_{J\,ik}\, T^{I\,*}_{J\,kj} \qquad (1)$$

Identical and non-identical particle states are treated in the same way.

The N/D method Chew, Mandelstam PR'60 is employed approximately for unitarizing T_J:

$$T_J = \frac{N}{D}\,,$$

where

$$\begin{aligned}
\mathrm{Im} D &= N\,\mathrm{Im}\,T_J^{-1} = -\rho N\,, \quad \text{for } s > 4m^2\,,\\
\mathrm{Im} D &= 0\,, \quad \text{for } s < 4m^2\,,\\
\mathrm{Im} N &= D\,\mathrm{Im}\,T_J\,, \quad \text{for } s < 0\,,\\
\mathrm{Im} N &= 0\,, \quad \text{for } s > 0\,,
\end{aligned}$$

due to the fact that the unitarity condition for the elastic channel $s > 4m^2$ is

$$\mathrm{Im}\,T_{IJ}^{-1} = -\rho\theta(s - s_{th})\,,$$

with $\rho = \sqrt{1 - 4m^2/s}/16\pi = q/8\pi\sqrt{s}$.

$$T_{IJ}(s) = \frac{Q(s)}{1 + g(s)Q(s)} \,,$$

Including LHC (e.g. in our perturbative calculation there is LHC from crossed exchange of resonances and crossed loops)

$$Q(s) \to N(s)$$

$N(s)$ only has LHC

$$T_{IJ}(s) = \frac{N(s)}{1 + g(s)\; N(s)} \,,$$

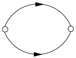
Matching with $T_J(s)|_{\chi PT} = T_2 + T_R + T_L$ up to one-loop at $\mathcal{O}(p^4)$:

$$T_2 + T_R + T_L = N(s) - N(s)g(s)N(s) + \mathcal{O}(\hbar^2)$$

$$N(s) = T_2 + T_R + T_L + N(s)g(s)N(s)$$

$\pi\pi$ elastic scattering. Integral equation for N_{JI}

Lacour, Oller, Meißner, ANP'10

Interaction kernel: N_{JI} Unitarity loop g: 

$T_{IJ} = \left[N_{IJ}^{-1} + g\right]^{-1}$:

$$\text{Im}N_{IJ} = \frac{|N_{IJ}|^2}{|T_{IJ}|^2}\text{Im}\,T_{IJ} = |1+gN_{IJ}|^2\text{Im}\,T_{IJ} \quad , \quad |\mathbf{p}|^2 < 0 \;.$$

$$N_{IJ}(A) = N_{IJ}(D) + \frac{A-D}{\pi}\int_{-\infty}^{0} dk^2 \frac{\text{Im}\,T_{IJ}(k^2)\,|1+g(k^2)N_{IJ}(k^2)|^2}{(k^2-A-i\epsilon)(k^2-D)}$$

The generalization to the inelastic case is straightforward:

$$T_{IJ}(s) = [1 + g(s) \cdot N_{IJ}(s)]^{-1} \cdot N_{IJ}(s)\,,$$

For $IJ = 00$ channel we have 5 channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$

$$N_0^0(s) = \begin{pmatrix} N_{\pi\pi\to\pi\pi} & N_{\pi\pi\to K\bar{K}} & N_{\pi\pi\to\eta\eta} & N_{\pi\pi\to\eta\eta'} & N_{\pi\pi\to\eta'\eta'} \\ N_{\pi\pi\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\eta\eta} & N_{K\bar{K}\to\eta\eta'} & N_{K\bar{K}\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta} & N_{K\bar{K}\to\eta\eta} & N_{\eta\eta\to\eta\eta} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta'} & N_{K\bar{K}\to\eta\eta'} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta'\to\eta\eta'} & N_{\eta\eta'\to\eta'\eta'} \\ N_{\pi\pi\to\eta'\eta'} & N_{K\bar{K}\eta'\eta'} & N_{\eta\eta\to\eta'\eta'} & N_{\eta\eta'\to\eta'\eta'} & N_{\eta'\eta'\to\eta'\eta'} \end{pmatrix}$$

$$g_0^0(s) = \begin{pmatrix} g_{\pi\pi} & 0 & 0 & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 & 0 & 0 \\ 0 & 0 & g_{\eta\eta} & 0 & 0 \\ 0 & 0 & 0 & g_{\eta\eta'} & 0 \\ 0 & 0 & 0 & 0 & g_{\eta'\eta'} \end{pmatrix}$$

For $IJ = 1\,0$ there are 3 channels: $\pi^0\eta$, $K\bar{K}$ and $\pi^0\eta'$

$$N(s)^1_0 = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to K\bar{K}} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\pi\eta'} \\ N_{\pi\eta\to\pi\eta'} & N_{K\bar{K}\to\pi\eta'} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix}$$

$$g(s)^1_0 = \begin{pmatrix} g_{\pi\eta} & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 \\ 0 & 0 & g_{\pi\eta'} \end{pmatrix}$$

For $IJ = 1/2\ 0$ and $1/2\ 1$ there are 3 channels: $K\pi$, $K\eta$ and $K\eta'$

$$N(s)_0^{1/2} = \begin{pmatrix} N_{K\pi\to K\pi} & N_{K\pi\to K\eta} & N_{K\pi\to K\eta'} \\ N_{K\pi\to K\eta} & N_{K\eta\to K\eta} & N_{K\eta\to K\eta'} \\ N_{K\pi\to K\eta'} & N_{K\eta\to K\eta'} & N_{K\eta'\to K\eta'} \end{pmatrix}$$

$$g(s)_0^{1/2} = \begin{pmatrix} g_{K\pi} & 0 & 0 \\ 0 & g_{K\eta} & 0 \\ 0 & 0 & g_{K\eta'} \end{pmatrix}$$

For the $IJ = 1\,1$ there are 2 channels

$$N_1^1(s) = \begin{pmatrix} N_{\pi\pi\to\pi\pi} & N_{\pi\pi\to K\bar{K}} \\ N_{\pi\pi\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} \end{pmatrix} ,$$

$$g_1^1(s) = \begin{pmatrix} g_{\pi\pi} & 0 \\ 0 & g_{K\bar{K}} \end{pmatrix} .$$

$IJ = 3/2\ 0$:

$$N(s)_0^{3/2} = N_{K\pi\to K\pi}$$
$$g(s)_0^{3/2} = g_{K\pi}$$

$IJ = 2\ 0$:

$$N(s)_0^2 = N_{\pi\pi\to\pi\pi} ,$$

$$g(s)_0^2 = g_{\pi\pi}$$

$IJ = 01$

$$N_1^0(s) = N_{K\bar{K}\to K\bar{K}} \,,$$

$$g_1^0(s) = g_{K\bar{K}} \,.$$

Numerical results

Observables fitted:

- $I = J = 0$: $\delta^{00}_{\pi\pi\to\pi\pi}$, $|S^{00}_{\pi\pi\to\pi\pi}|$, $\frac{1}{2}|S^{00}_{\pi\pi\to K\bar{K}}|$, $\delta^{00}_{\pi\pi\to K\bar{K}}$
- $I = J = 1$: $\delta^{11}_{\pi\pi\to\pi\pi}$
- $I = 1/2\ J = 0\ , 1$: $\delta^{\frac{1}{2}0}_{\pi K\to\pi K}$, $\delta^{\frac{1}{2}1}_{\pi K\to\pi K}$
- $I = 2\ J = 0$: $\delta^{20}_{\pi\pi\to\pi\pi}$
- $I = 3/2\ J = 0$: $\delta^{\frac{3}{2}0}_{\pi K\to\pi K}$
- $I = 1\ J = 0$: $\pi\eta$ event distribution around $a_0(980)$

$$\frac{dN_{\pi\eta}}{dE_{\pi\eta}} = q_{\pi\eta}\,\mathcal{N}\Big|\ T_{K\bar{K}\to\pi\eta}(s) + c\ T_{\pi\eta\to\pi\eta}(s)\ \Big|^2 .$$

- m_η, $m_{\eta'}$

Subtraction Constants: The number of free ones can be reduced by applying Isospin and SU(3) symmetry. We also assume nonet symmetry in some cases. Jido,Oller,Oset,Ramos,Meißner, NPA'03

• Isospin Symmetry requires that all the a_{SL}^{IJ} are the same separately for $\pi\pi$, $K\bar{K}$ and $K\pi$

• Nonet Symmetry requires that all a_{SL}^{IJ} are the same for a given J

$$a_{SL}^{\frac{1}{2}0\,,\,K\pi} = a_{SL}^{\frac{3}{2}0\,,\,K\pi}$$
$$a_{SL}^{00\,,\,K\bar{K}} = a_{SL}^{10\,,\,K\bar{K}} = a_{SL}^{00\,,\,\eta\eta} = a_{SL}^{00\,,\,\eta\eta'} = a_{SL}^{00\,,\,\eta'\eta'} = a_{SL}^{10\,,\,\pi\eta'}$$
$$a_{SL}^{00\,,\,\pi\pi} = a_{SL}^{20\,,\,\pi\pi}$$

All the subtraction constants in the vector channels are set equal to $a_{SL}^{00\,,\,\pi\pi}$ (play a little role).
Unitarization is much less important for these channels.

Large N_C relation for the singlet scalar couplings:

$$\widetilde{c}_d = \frac{1}{\sqrt{3}} c_d \ , \ \widetilde{c}_m = \frac{1}{\sqrt{3}} c_m$$

14 relevant free parameters:

$$c_d = (20^{+2}_{-5})\,\mathrm{MeV}\,, \qquad c_m = (42^{+4}_{-9})\,\mathrm{MeV}\,,$$
$$M_{S_8} = (1400^{+70}_{-60})\,\mathrm{MeV}\,, \qquad M_{S_1} = (1100^{+30}_{-60})\,\mathrm{MeV}\,,$$
$$G_V = (62.1^{+1.9}_{-2.1})\,\mathrm{MeV}\,, \qquad a_{SL}^{1\,0\,,\pi\eta} = 2.0^{+3.3}_{-4.5}\,,$$
$$a_{SL}^{00\,,\pi\pi} = -1.27^{+0.12}_{-0.12}\,, \qquad a_{SL}^{00\,,K\bar{K}} = -0.95^{+0.33}_{-0.16}\,,$$
$$a_{SL}^{\frac{1}{2}\,0\,,K\pi} = -1.12^{+0.12}_{-0.17}\,, \qquad a_{SL}^{\frac{1}{2}\,0\,,K\eta} = -0.08^{+0.38}_{-1.04}\,,$$
$$a_{SL}^{\frac{1}{2}\,0\,,K\eta'} = -1.25^{+1.11}_{-1.23}\,, \qquad \delta L_8 = (0.23^{+0.29}_{-0.19}) \times 10^{-3}\,,$$
$$M_0 = (950^{+50}_{-50})\,\mathrm{MeV}\,, \qquad \Lambda_2 = -0.37^{+0.19}_{-0.19}\,,$$

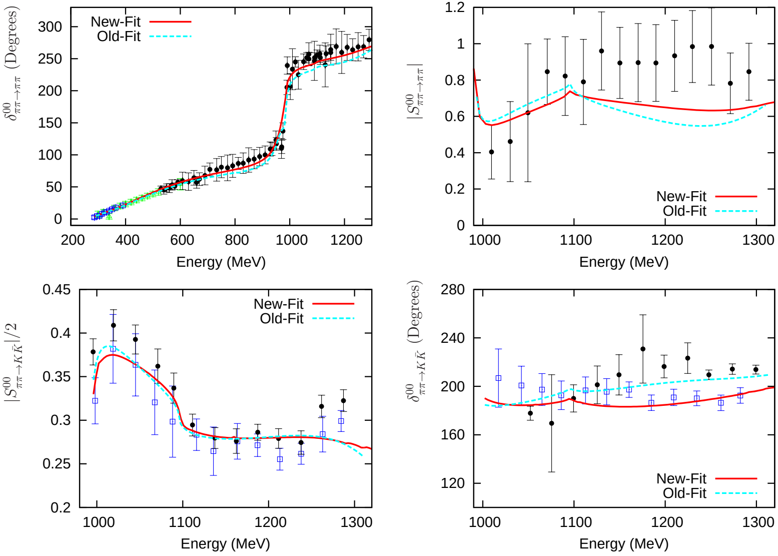
with $\chi^2/\mathrm{d.o.f} = 714/(348-16) \simeq 2.15$.
$n_\sigma = \Delta\chi^2/\sqrt{2\chi^2} \leq 2$ to get the errors, $n_\sigma = 2$ Etkin *et al.* PRD'82
$G_V = 55$ MeV Gasser and Leutwyler '85

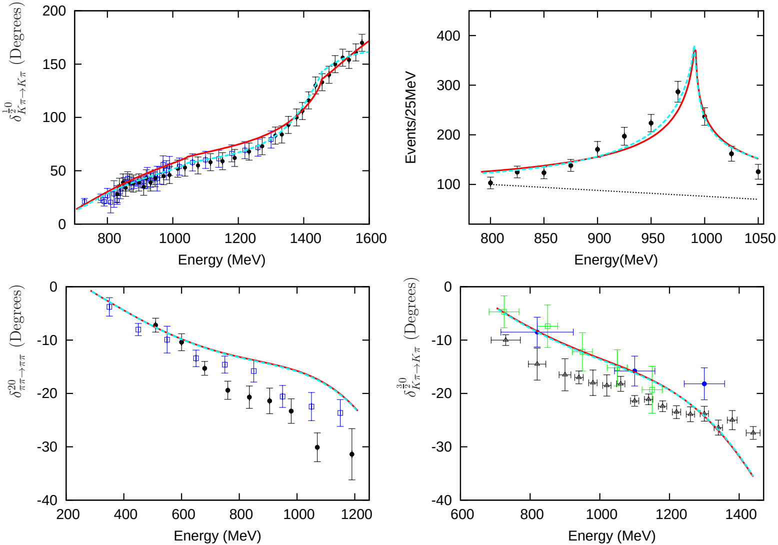
- Large N_c $U(3)$ nonet symmetry:
  - $a^{00}_{SL} \simeq a^{\frac{1}{2}0}_{SL}$
  - $M_8 = M_1$ ($\sim 30\%$ discrepancy)
  - M_8 and M_1 in good agreement with previous determinations [I,II]
  - $c_d = (20^{+2}_{-5})$ and $c_m = (42^{+4}_{-9})$ MeV are compatible with previous determinations [I] $c_d = 19.1^{+2.4}_{-2.1}$; [II] $c_d \simeq 24$; [III] $c_d = 26 \pm 7$. c_m always with large errors.

[I] Oller, Oset PRD'99;[II] Jamin, Oller, Pich NPB'00; [III] Guo, Sanz-Cillero PRD'09

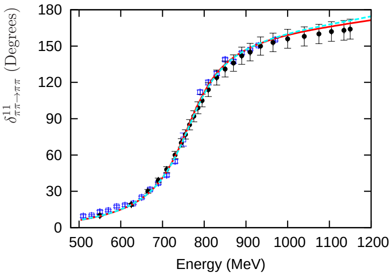
Our results	Bijnens, Jemos NPB'12
$L_4 = 0.09 \pm 0.04 \times 10^{-3}$	$L_4 = 0.75 \pm 0.75 \times 10^{-3}$
$L_5 = 0.67^{+0.03}_{-0.19} \times 10^{-3}$	$L_5 = 0.58 \pm 0.13 \times 10^{-3}$

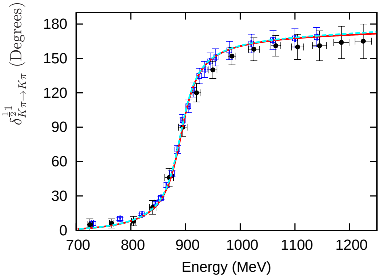
  - We can obtain good fits with 6 free parameters less.


New-Fit
Old-Fit
$\delta^{00}_{\pi\pi\to\pi\pi}$ (Degrees)
Energy (MeV)
$|S^{00}_{\pi\pi\to\pi\pi}|$
$|S^{00}_{\pi\pi\to K\bar{K}}|/2$
$\delta^{00}_{\pi\pi\to K\bar{K}}$ (Degrees)



$\delta^{\frac{1}{2}0}_{K\pi\to K\pi}$ (Degrees)
Energy (MeV)
Events/25MeV
Energy(MeV)
$\delta^{20}_{\pi\pi\to\pi\pi}$ (Degrees)
Energy (MeV)
$\delta^{\frac{3}{2}0}_{K\pi\to K\pi}$ (Degrees)
Energy (MeV)





• We need to distinguish between the $\rho(770)$ and $K^*(892)$ bare masses in order to fit the phase shifts accurately:

$$M_\rho = (801.0^{+7.0}_{-7.5})\,\mathrm{MeV}\,, \qquad M_{K^*} = (909.0^{+7.5}_{-6.9})\,\mathrm{MeV}$$

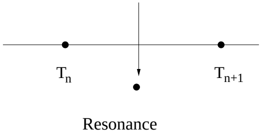
- Unitarity (vector self-energies due to two-pseudoscalar loops) does not give the necessary shift to the vector octet bare mass.

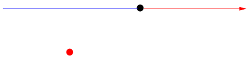
- Unitarity provides the right width for every vector resonance.

Poles from the unitarized amplitudes

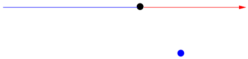
By crossing the real axis between the thresholds $T_n < s < T_{n+1}$ one directly reaches the Riemann sheet

$$(\underbrace{-,-,..-}_{n},+,+,\ldots)$$





Cusp Rise Left. Resonance Fall Right.
Asymmetric shape.



Resonance Rise Right. Cusp Fall Right.
Asymmetric shape.

R	M (MeV)	$\Gamma/2$ (MeV)	$\|\text{Residues}\|^{1/2}$ (GeV)	Ratios	
σ	441	247	3.02 $(\pi\pi)$	0.50 $(K\bar{K}/\pi\pi)$	0.17 $(\eta\eta/\pi\pi)$
				0.33$(\eta\eta'/\pi\pi)$	0.11$(\eta'\eta'/\pi\pi)$
$f_0(980)$	977	29	1.8$(\pi\pi)$	2.6$(K\bar{K}/\pi\pi)$	1.6$(\eta\eta/\pi\pi)$
				1.0$(\eta\eta'/\pi\pi)$	0.7$(\eta'\eta'/\pi\pi)$
$f_0(1370)$(iii)	1359	169	3.2$(\pi\pi)$	1.0$(K\bar{K}/\pi\pi)$	1.2$(\eta\eta/\pi\pi)$
				1.5$(\eta\eta'/\pi\pi)$	0.7$(\eta'\eta'/\pi\pi)$
$f_0(1370)$(v)	1496	152	2.2$(\pi\pi)$	1.4$(K\bar{K}/\pi\pi)$	1.2$(\eta\eta/\pi\pi)$
				2.9$(\eta\eta'/\pi\pi)$	1.7$(\eta'\eta'/\pi\pi)$
κ	641	304	4.8$(K\pi)$	0.86$(K\eta/K\pi)$	0.747$(K\eta'/K\pi)$
$K_0^*(1430)$(iii)	1483	132	4.4$(K\pi)$	0.33 $(K\eta/K\pi)$	1.22$(K\eta'/K\pi)$
$a_0(980)$	1006	22	2.45$(\pi\eta)$	1.9 $(K\bar{K}/\pi\eta)$	0.03$(\pi\eta'/\pi\eta)$
$a_0(1450)$	1460	174	4.5$(\pi\eta)$	0.36$(K\bar{K}/\pi\eta)$	$0.97^{+0.2}_{-0.1}(\pi\eta'/\pi\eta)$
$\rho(770)$	760	71	2.44$(\pi\pi)$	0.64$(K\bar{K}/\pi\pi)$	
$K^*(892)$	892	25	1.85$(K\pi)$	0.91$(K\eta/K\pi)$	0.41$(K\eta'/K\pi)$
$\phi(1020)$	1019	1.9	0.85$(K\bar{K})$		

- σ or $f_0(600)$, $IJ = 0\,0$ $(-,+,+,+,+)$

$$M_\sigma = 440^{+3}_{-3}\,\mathrm{MeV}\,, \quad \Gamma_\sigma/2 = 258 \pm 12\,\mathrm{MeV}\,,$$

$$|g_{\sigma\pi\pi}| = 3.02^{+0.03}_{-0.03}\,\mathrm{GeV}\,,$$

$$|g_{\sigma K\bar{K}}|/|g_{\sigma\pi\pi}| = 0.51^{+0.03}_{-0.02}\,,\ |g_{\sigma\eta\eta}|/|g_{\sigma\pi\pi}| = 0.06^{+0.03}_{-0.01}$$

$$|g_{\sigma\eta\eta'}|/|g_{\sigma\pi\pi}| = 0.16^{+0.03}_{-0.02}\,,\ |g_{\sigma\eta'\eta'}|/|g_{\sigma\pi\pi}| = 0.05^{+0.03}_{-0.03}$$

$$T \to -\frac{g_i\, g_j}{s - M^2}\quad , \quad g^2_{\sigma\pi\pi} = \frac{-1}{2\pi i}\oint_{|s-s_\sigma|=R} T^{\mathrm{II}}(s)\, ds\,.$$

$M_\sigma = 441^{+16}_{-8}$, $\Gamma_\sigma/2 = 272^{+9}_{-13}$ Caprini *et al.* PRL'06
$M_\sigma = 484 \pm 17$, $\Gamma_\sigma/2 = 255 \pm 10$ García-Martín *et al.* PRD'07
$M_\sigma = 456 \pm 6$, $\Gamma_\sigma/2 = 241 \pm 17$ Albaladejo, Oller PRL'08

Narison *et al.* PLB'10 interpret the large $K\bar{K}$ coupling of the σ as an indication for a glueball nature.
We also have such a large coupling. In our case it corresponds to a dynamically generated resonance.

- $f_0(980)$, $IJ = 00$ $\quad (-,+,+,+,+)$

$$M_{f_0} = 981^{+9}_{-7}\,\text{MeV}\,, \quad \Gamma_{f_0}/2 = 22^{+5}_{-7}\,\text{MeV}\,,$$
$$|g_{f_0\pi\pi}| = 1.7^{+0.3}_{-0.3}\,\text{GeV}$$
$$|g_{f_0K\bar{K}}|/|g_{f_0\pi\pi}| = 2.3^{+0.3}_{-0.2}\,,\ |g_{f_0\eta\eta}|/|g_{f_0\pi\pi}| = 1.6^{+0.3}_{-0.3}$$
$$|g_{f_0\eta\eta'}|/|g_{f_0\pi\pi}| = 1.2^{+0.1}_{-0.2}\,,\ |g_{f_0\eta'\eta'}|/|g_{f_0\pi\pi}| = 0.7^{+0.4}_{-0.5}$$

- $f_0(1370)$, $IJ = 00$ $\quad (-,-,-,+,+)$

$$M_{f_0} = 1401^{+58}_{-37}\,\text{MeV}\,, \quad \Gamma_{f_0}/2 = 106^{+36}_{-23}\,\text{MeV}\,,$$
$$|g_{f_0\pi\pi}| = 2.4^{+0.2}_{-0.1}\,\text{GeV}$$
$$|g_{f_0K\bar{K}}|/|g_{f_0\pi\pi}| = 0.62^{+0.04}_{-0.05}\,,\ |g_{f_0\eta\eta}|/|g_{f_0\pi\pi}| = 0.9^{+0.1}_{-0.1}$$
$$|g_{f_0\eta\eta'}|/|g_{f_0\pi\pi}| = 1.7^{+0.4}_{-0.6}\,,\ |g_{f_0\eta'\eta'}|/|g_{f_0\pi\pi}| = 1.1^{+0.4}_{-0.5}$$

Both resonances have strong couplings to states with η, η'

- κ or $K_0^*(800)$, $IJ = 1/2\,0$, $(-,+,+)$

$$M_\kappa = 665^{+9}_{-9}\,\mathrm{MeV}\,, \quad \Gamma_\kappa/2 = 268^{+21}_{-6}\,\mathrm{MeV}\,,$$
$$|g_{\kappa K\pi}| = 4.2^{+0.2}_{-0.2}\,\mathrm{GeV}$$
$$|g_{\kappa K\eta}|/|g_{\kappa K\pi}| = 0.7^{+0.1}_{-0.1}\,,\ |g_{\kappa K\eta'}|/|g_{\kappa K\pi}| = 0.50^{+0.1}_{-0.1}$$

Pole $\sqrt{s} = (658 \pm 13 - i\,278 \pm 12)$ MeV Descotes, Moussallam EPJC'06

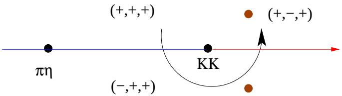
- $K_0^*(1430)$, $IJ = 1/2\,0$, $(-,-,+)$

$$M_{K_0^*} = 1428^{+56}_{-23}\,\mathrm{MeV}\,, \quad \Gamma_{K_0^*}/2 = 87^{+53}_{-28}\,\mathrm{MeV}\,,$$
$$|g_{K_0^* K\pi}| = 3.3^{+0.5}_{-0.4}\,\mathrm{GeV}$$
$$|g_{K_0^* K\eta}|/|g_{K_0^* K\pi}| = 0.54^{+0.07}_{-0.02}\,,\ |g_{K_0^* K\eta'}|/|g_{K_0^* K\pi}| = 1.2^{+0.2}_{-0.3}$$

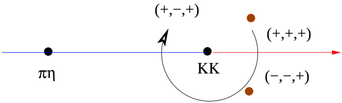
There is also another relevant pole in $(-,-,-)$ at $\sqrt{s} = 1352^{+74}_{-29} - i\,114^{+60}_{-39}$. Asymmetric form.

- $a_0(980)$, $IJ = 10$, $(+,-,+)$

$$M_{a_0} = 1012^{+25}_{-7}\,\mathrm{MeV}\,,\ \Gamma_{a_0}/2 = 16^{+50}_{-13}\,\mathrm{MeV}\,,$$
$$|g_{a_0\pi\eta}| = 2.5^{+1.3}_{-0.8}\,\mathrm{GeV}$$
$$|g_{a_0K\bar{K}}|/|g_{a_0\pi\eta}| = 1.9^{+0.2}_{-0.3}\ ,\ \ |g_{a_0\pi\eta'}|/|g_{a_0\pi\eta}| = 0.01^{+0.03}_{-0.01}$$



Below KK threshold



Above KK threshold

Vanishing coupling to $\pi\eta'$. This channel plays a negligible role in the $a_0(980)$ energy region.

- $a_0(1450)$, $IJ = 1\,0$, $(-,-,-)$

$$M_{a_0} = 1368^{+68}_{-68}\,\text{MeV}\,,\ \Gamma_{a_0}/2 = 71^{+48}_{-23}\,\text{MeV}\,,$$
$$|g_{a_0\pi\eta}| = 2.3^{+0.4}_{-0.5}\,\text{GeV}$$
$$|g_{a_0K\bar{K}}|/|g_{a_0\pi\eta}| = 0.6^{+0.7}_{-0.2}\ ,\ |g_{a_0\pi\eta'}|/|g_{a_0\pi\eta}| = 0.6^{+0.2}_{-0.1}$$

PDG: $\Gamma = 265 \pm 13$ MeV. We are lacking important decay channels: $\omega\pi\pi$, $a_0(980)\pi\pi$

- $\rho(770)$, $IJ = 1\,1$ $(-,+)$

$$M_\rho = 762^{+4}_{-4}\,\text{MeV}\,,\quad \Gamma_\rho/2 = 72^{+2}_{-2}\,\text{MeV}\,,$$
$$|g_{\rho\pi\pi}| = 2.48^{+0.03}_{-0.05}\,\text{GeV}\ ,\ |g_{\rho K\bar{K}}|/|g_{\rho\pi\pi}| = 0.64^{+0.01}_{-0.01}$$

- $K^*(892)$, $IJ = 1/2\,1$ $(-,+,+)$

$$M_{K^*} = 891^{+3}_{-4}\,\text{MeV}\,,\quad \Gamma_{K^*}/2 = 25^{+2}_{-1}\,\text{MeV}\,,$$
$$|g_{K^*\pi K}| = 1.86^{+0.05}_{-0.05}\,\text{GeV}$$
$$|g_{K^*K\eta}|/|g_{K^*K\pi}| = 0.91^{+0.03}_{-0.02}\ ,\ |g_{K^*K\eta'}|/|g_{K^*K\pi}| = 0.45^{+0.08}_{-0.08}$$

SS, PP correlators. Sum rules

$$\delta^{ab}\Pi_R(p^2) = i\int d^4x\, e^{ip\cdot x} < 0|T[R^a(x)R^b(0)]|0 > ,$$

In the chiral limit we have the second spectral function sum rules:

$$\int_0^\infty \left[\mathrm{Im}\,\Pi_R(s) - \mathrm{Im}\,\Pi_{R'}(s)\right] ds = 0\,,$$

R^a refers to S^a or P^a with $a = 0,\ 1,\ 3$ or 8.

Splitting between hadronic and high energy regions:

$$\int_0^{s_0} \left[\mathrm{Im}\,\Pi_R(s) - \mathrm{Im}\,\Pi_{R'}(s)\right] ds + \int_{s_0}^\infty \left[\mathrm{Im}\,\Pi_R(s) - \mathrm{Im}\,\Pi_{R'}(s)\right] ds = 0$$

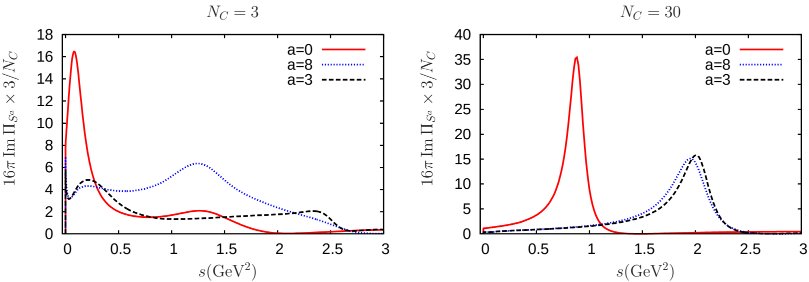
$s_0 = 2.5,\ 3,\ 3.5\ \mathrm{GeV}^2$

For $s > s_0$ we employ the OPE expression (or theoretical spectral function) from Jamin,Münz ZPC'93 calculated up to $\mathcal{O}(\alpha_s)$ and including up to dimension 5 operators.

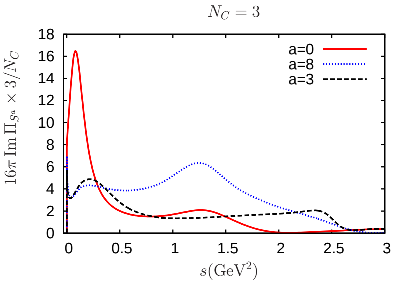
The theoretical spectral functions cancel each other in the chiral limit. The second integral for $s > s_0$ is 0

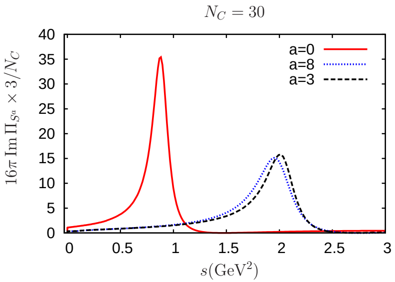
$$
\begin{aligned}
W_i &= 16\pi \int_0^{s_0} \mathrm{Im}\, \Pi_i(s)\, ds\,, \quad i = S^8, S^0, S^3, P^0, P^8, P^3\,, \\
\overline{W} &= \frac{\sum_{i=(S^8,S^0,S^3,P^0,P^8,P^3)} W_i}{3 \times 6}\,, \\
\sigma &= \sum_{i=(S^8,S^0,S^3,P^0,P^8,P^3)} \sqrt{\frac{(W_i - \overline{W})^2}{17}}\,,
\end{aligned}
$$

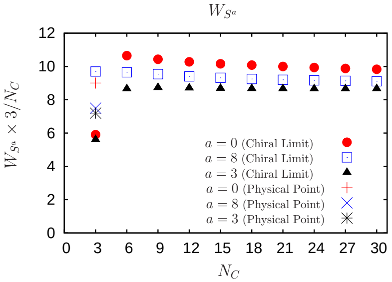
	W_{S0}	W_{S8}	W_{S3}	W_{P0}	W_{P8}	W_{P3}	$\overline{W}$	σ	$\sigma/\overline{W}$
Physical masses	8.6 9.0 9.6	7.4 7.5 7.7	7.0 7.2 7.4	8.9	11.3	10.1	9.0	1.5	0.16
$m_q = 0$	5.8 5.9 6.1	9.6 9.7 9.7	5.4 5.6 5.7	5.5	7.5	7.5	6.9	1.5	0.22

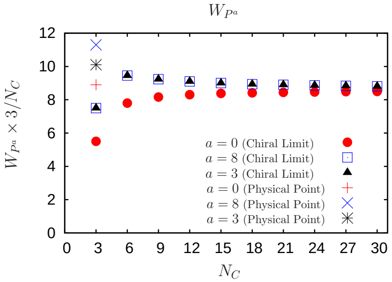


- The equality of the integrals for the spectral functions is satisfied at the level of 20%
- The lightest peak for $a = 0$ is the $f_0(600)$
- It is interesting the movement of the peaks with N_C
- For $N_C \to \infty$ only the bare poles remains

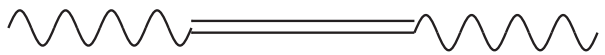








In the large N_C limit the SS, PP correlators reduce to



$c_m^2 = \frac{F^2}{8} + d_m^2$
perfectly fulfilled by
$c_m = 42$ MeV from our fit

Calculation of the spectral functions

Scalar Form Factors

$$\mathrm{Im}\Pi_{S^a}(s) = \sum_i \rho_i(s)\, F_i^a(s)\, F_i^{a*}(s)\, \theta(s - s_i^{\mathrm{th}})$$

$$\rho_i(s) = \frac{\sqrt{[s-(m_P+m_Q)^2][s-(m_P-m_Q)^2]}}{16\pi\, s} = \frac{q_i}{8\pi\sqrt{s}}$$

$$F_i^a(s) = \frac{1}{B}\langle\, 0|\, \bar{q}\frac{\lambda_a}{\sqrt{2}} q\,|\,(PQ)_i\,\rangle$$

$$\mathrm{Im}\, F_j^I(s) = \sum_{k=1}^{Z} T_{jk}^{IJ}(s)^*\, \rho_k(s)\, F_k^I(s)$$

Resumming of the right hand cut

$$T_{IJ}(s) = \left[1 + N_{IJ}(s)\, g_{IJ}(s)\right]^{-1} N_{IJ}(s)$$

$$F_I(s) = \left[1 + N_{IJ}(s)\, g_{IJ}(s)\right]^{-1} R_I(s)$$

$$= \left[1 - N_{IJ}(s)\, g_{IJ}(s)\right] R_I(s) + \mathcal{O}(g^2)$$

with

$$N_{IJ}(s) = T_{IJ}(s)^{(2)+\text{Res}+\text{Loop}} + T_{IJ}(s)^{(2)}\, g_{IJ}(s)\, T_{IJ}(s)^{(2)}\,,$$

$$R_I(s) = F^I(s)^{(2)+\text{Res}+\text{Loop}} + N_{IJ}(s)^{(2)}\, g_{IJ}(s)\, F_I(s)^{(2)}\,.$$

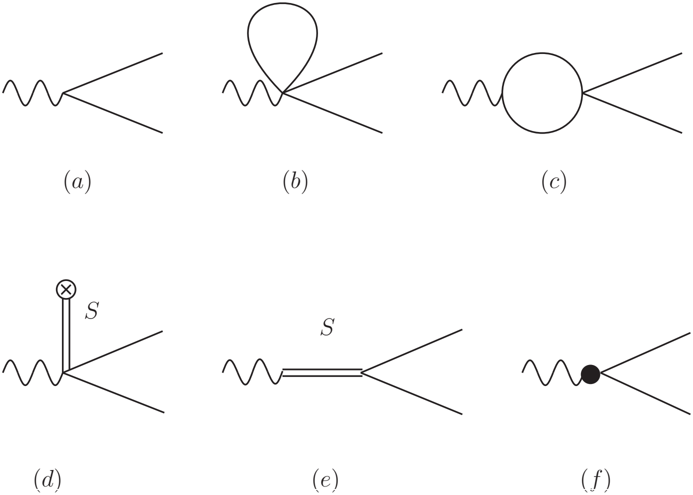
$R_I(s)$ has no cuts

Meißner, JAO NPA'01

It is determined perturbatively in Resonance $U(3)$ ChPT

$F_I(s)$: One-loop Resonance $U(3)$ ChPT calculation:

Guo, JAO forthcoming



Scalar Radius of the pion

$$F_{\pi\pi}^{\bar{u}u+\bar{d}d}(s) = F_{\pi\pi}^{\bar{u}u+\bar{d}d}(0)\left[1+\frac{1}{6}\langle\, r^2\,\rangle_S^\pi\, s + \ldots\right]$$

$$\langle\, r^2\,\rangle_S^\pi = 0.5\,\mathrm{fm}^2$$

Dispersive studies:
$\langle\, r^2\,\rangle_S^\pi \simeq 0.6\,\mathrm{fm}^2$

Colangelo, Gasser, Leutwyler NPB'01; Roca, JAO PLB'07

Pseudoscalar Form Factors: $\mathrm{Im}\,\Pi_{P^a}(s) = \sum_i \pi\,\delta(s - m_{P_i}^2)\,|H_i^a(s)|^2$

Semi-local duality

Relation between of s-channel dynamics (including sum of resonances) and t-channel Regge pole exchanges

For energies not high the duality works at the average level

Peláez, Pennington, Ruiz de Elvira, Wilson PRD'11 **[I]** applied average duality for studying the $f_0(600)$ and its non-standard nature

They applied the Inverse Amplitude Method (IAM) for unitarizing ChPT at the two-loop level and paid attention to the $\rho(770)$ and $f_0(600)$ resonances

Fixed-t scattering, $\nu = (s - u)$

We use the same parameterizations as Peláez *et al.* PRD'11 for $\mathrm{Im}\, A^{(I)}_{t,\mathrm{Regge}}(\nu, t)$ that contain the ρ trajectory for $I_t = 1$ and the Pomeron for $I_t = 0$. No isotensor Regge contribution.

$$\mathrm{Im}\, A^{(I)}_{s,\mathrm{Hadrons}}(\nu, t) = \sum_J (2J + 1)\, \mathrm{Im}\, A^{(I)}_J(s)\, P_J(z_s)\,,$$

$\mathrm{Im}\, A^{(I)}_J(s)$ are the partial wave that we already calculated and studied.

$$R^I_n = \frac{\int_{\nu_1}^{\nu_2} \nu^{-n}\, \mathrm{Im}\, A^{(I)}_t(\nu, t)}{\int_{\nu_1}^{\nu_3} \nu^{-n}\, \mathrm{Im}\, A^{(I)}_t(\nu, t)}\,,$$

it is calculated with Regge theory and in terms of hadronic degrees of freedom. The results are compared.
ν_1 =threshold, $\nu_2 = 1$ GeV^2 and $\nu_3 = 2$ GeV^2.

We take $n = 0, 1, 2, 3$, as n increases the integrals are more and more sensitive to the low energy region.

As n increases one is increasingly sensitive to the splitting in mass between the lightest resonances

	n	$\mathbf{R_n^0}$ $t = t_{\rm th}$	$\mathbf{R_n^0}$ $t = 0$	$\mathbf{R_n^1}$ $t = t_{\rm th}$	$\mathbf{R_n^1}$ $t = 0$	F_n^{21} $t = t_{\rm th}$ $\nu_{\mathbf{max}} =$	F_n^{21} $t = 0$ 2 GeV^2
Regge	0	0.225	0.233	0.325	0.353	~ 0	~ 0
	1	0.425	0.452	0.578	0.642	~ 0	~ 0
	2	0.705	0.765	0.839	0.908	~ 0	~ 0
	3	0.916	0.958	0.966	0.990	~ 0	~ 0
Ours	0	0.669	0.628	0.836	0.817	-0.113	0.040
$S + P$	1	0.837	0.812	0.919	0.908	-0.230	-0.087
Waves	2	0.934	0.924	0.966	0.962	-0.129	0.028
	3	0.979	0.976	0.989	0.988	0.169	0.345
Ours	0	0.410	0.400	0.453	0.468	0.531	0.587
$S + P + D$	1	0.653	0.643	0.694	0.706	0.154	0.236
Waves	2	0.850	0.844	0.875	0.882	0.027	0.155
	3	0.954	0.953	0.965	0.968	0.225	0.388

D-wave

We consider here the exchange of a nonet of tensor resonances employing the Lagrangians of Ecker, Zaune, EPJC'07 extended to $U(3)$

These exchanges contribute to the N_{IJ} function

$$\mathcal{L}_T = -\frac{1}{2}\langle\, T_{\mu\nu} D_{\mathrm{T}}^{\mu\nu,\rho\sigma} T_{\rho\sigma}\,\rangle + g_T \langle\, T_{\mu\nu}\{u_\mu, u_\nu\}\,\rangle + \beta\langle\, T^\mu_\mu u_\nu u^\nu\,\rangle + \gamma\langle\, T^\mu_\mu \chi_+\,\rangle\,,$$

$$T_{\mu\nu} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^1}{\sqrt{3}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^1}{\sqrt{3}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} + \frac{f_2^1}{\sqrt{3}} \end{pmatrix}_{\mu\nu} .$$

Ideal mixing:

$$f_2(1270) = \sqrt{\frac{2}{3}} f_2^1 + \sqrt{\frac{1}{3}} f_2^8\,, \quad f_2' = \sqrt{\frac{1}{3}} f_2^1 - \sqrt{\frac{2}{3}} f_2^8\,.$$

Short distance constraints: forward elastic partial waves satisfy once subtracted dispersion relation. We generalize them to the $U(3)$ case Guo,JAO forthcoming.

They are important to obtain meaningful results once tensor resonances are included

Several $\mathcal{O}(p^4)$ monomials are needed. (The results are then independent of β but $\beta = -g_T$ avoids including $\mathcal{O}(p^6)$ monomials.)

$$F_n^{II'} = \frac{\int_{\nu_1}^{\nu_{\mathbf{max}}} \nu^{-n} \, \mathrm{Im}\, A_t^{(I)}(\nu, t)}{\int_{\nu_1}^{\nu_{\mathbf{max}}} \nu^{-n} \, \mathrm{Im}\, A_t^{(I')}(\nu, t)}$$

$\nu_{max} = 1$ or 2 GeV^2.
Most interesting $I = 2$ and $I = 1$

$$A_t^{(1)}(s,t) = \frac{1}{3}A_s^{(0)}(s,t) + \frac{1}{2}A_s^{(1)}(s,t) - \frac{5}{6}A_s^{(2)}(s,t)$$
$$A_t^{(2)}(s,t) = \frac{1}{3}A_s^{(0)}(s,t) - \frac{1}{2}A_s^{(1)}(s,t) + \frac{1}{6}A_s^{(2)}(s,t)$$

$F_n^{21} \to -1$ if scalar contribution is dropped
$F_n^{21} \to +1$ if vector contribution is dropped

	n	R_n^0 $t=t_{\rm th}$	R_n^0 $t=0$	R_n^1 $t=t_{\rm th}$	R_n^1 $t=0$	$\mathbf{F_n^{21}}$ $t=t_{\rm th}$ $\nu_{\bf max}=$	$\mathbf{F_n^{21}}$ $t=0$ 2 GeV^2
Regge	0	0.225	0.233	0.325	0.353	~ 0	~ 0
	1	0.425	0.452	0.578	0.642	~ 0	~ 0
	2	0.705	0.765	0.839	0.908	~ 0	~ 0
	3	0.916	0.958	0.966	0.990	~ 0	~ 0
Ours	0	0.669	0.628	0.836	0.817	-0.113	0.040
$S+P$	1	0.837	0.812	0.919	0.908	-0.230	-0.087
Waves	2	0.934	0.924	0.966	0.962	-0.129	0.028
	3	0.979	0.976	0.989	0.988	0.169	0.345
Ours	0	0.410	0.400	0.453	0.468	0.531	0.587
$S+P+D$	1	0.653	0.643	0.694	0.706	0.154	0.236
Waves	2	0.850	0.844	0.875	0.882	0.027	0.155
	3	0.954	0.953	0.965	0.968	0.225	0.388

The numbers in absolute value are typically much smaller than unity. Average duality is fulfilled quite accurately

	n	R_n^0 $t=t_{\rm th}$	R_n^0 $t=0$	R_n^1 $t=t_{\rm th}$	R_n^1 $t=0$	$\mathbf{F_n^{21}}$ $t=t_{\rm th}$ $\nu_{\mathbf{max}}=$	$\mathbf{F_n^{21}}$ $t=0$ 2 GeV^2
Regge	0	0.225	0.233	0.325	0.353	~ 0	~ 0
	1	0.425	0.452	0.578	0.642	~ 0	~ 0
	2	0.705	0.765	0.839	0.908	~ 0	~ 0
	3	0.916	0.958	0.966	0.990	~ 0	~ 0
Ours	0	0.669	0.628	0.836	0.817	-0.113	0.040
$S+P$	1	0.837	0.812	0.919	0.908	-0.230	-0.087
Waves	2	0.934	0.924	0.966	0.962	-0.129	0.028
	3	0.979	0.976	0.989	0.988	0.169	0.345
Ours	0	0.410	0.400	0.453	0.468	0.531	0.587
$S+P+D$	1	0.653	0.643	0.694	0.706	0.154	0.236
Waves	2	0.850	0.844	0.875	0.882	0.027	0.155
	3	0.954	0.953	0.965	0.968	0.225	0.388

For $n=0$, the D waves make a too large contribution. They overbalance the $\rho(770)$. The $\rho(1450)$ gives a negligible contribution because its BR to $\pi\pi$ ($\simeq 6\%$) is too small [PDG]. Thus, heavier vector resonances are needed. For larger n they have no impact due to their large mass.

Extrapolating in N_C

Due to the uncertainties in the large N_C values of the resonance parameters we consider several options:

Scenario 1: Leading running with N_C

$$\{c_d(N_C), c_m(N_C), \widetilde{c}_d(N_C), \widetilde{c}_m(N_C), G_V(N_C)\}$$
$$= \{c_d(3), c_m(3), \widetilde{c}_d(3), \widetilde{c}_m(3), G_V(3)\} \times \sqrt{\frac{N_C}{3}} \,,$$
$$\{M_{S_1}(N_C), M_{S_8}(N_C), M_\rho(N_C), M_{K^*}(N_C), M_\omega(N_C), M_\phi(N_C), a_{SL}(N_C)\}$$
$$= \{M_{S_1}(3), M_{S_8}(3), M_\rho(3), M_{K^*}(3), M_\omega(3), M_\phi(3), a_{SL}(3)\}$$

For the other cases subleading $1/N_C$ corrections are introduced as:

$$A(N_C) = \alpha + \beta \frac{1}{N_C}$$
$$\alpha = A(\infty) \quad , \quad \beta = 3\,[A(3) - A(\infty)]$$

Scenario 2: Large N_C expression for G_V

$$G_V(N_C) = \frac{F_\pi(N_C)}{\sqrt{3}}$$

High energy constraint of pion vector f.f. at one-loop level Pich,Rosell,Sanz-Cillero JHEP'109, radiative τ decays Guo, Roig PRD'113016, $\pi\pi$ scattering Guo,JAO PRD'11; Guo, Sanz-Cillero, Zheng JHEP'07, model for $\pi\pi$ scattering in extra dimensions Chivukula *et al.* PRD'07

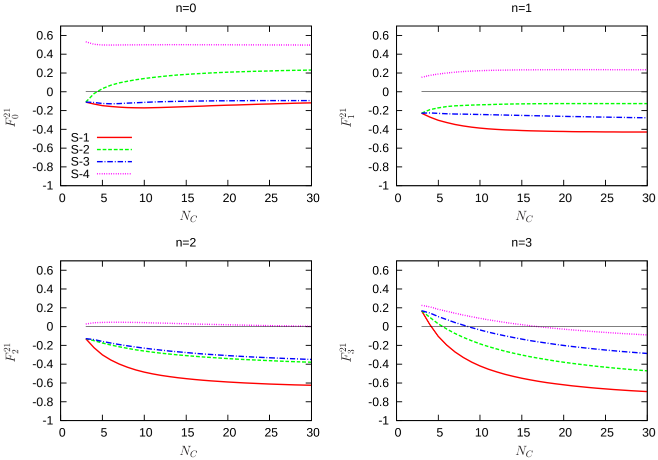
$$G_V(N_C) = G_V(N_C=3)\sqrt{\frac{N_C}{3}} \times \left[1 + \frac{G_V(N_C=3) - G_V^{\rm Nor}(N_C\to\infty)}{G_V(N_C=3)}\left(\frac{3}{N_C}-1\right)\right]$$

$$G_V^{\rm Nor}(N_C\to\infty) = G_V(N_C\to\infty)\sqrt{\frac{3}{N_C}}$$

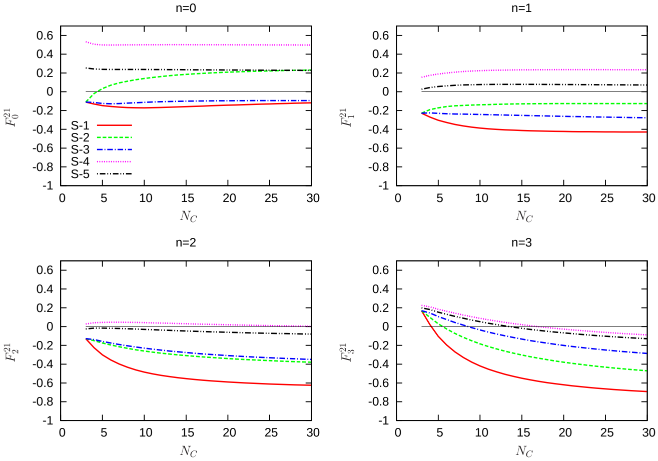
Scenario 3: Large N_C degenerate masses for $\rho(770)$ and $S_1(1000)$
$M(\infty) = 930$ MeV
Mean between the bare ρ and S_1 masses
Very close to the $N_C \to \infty$ ρ mass of [I]

$$M^2(N_C) = M^2(N_C = 3)\left[1 + \frac{M^2(N_C = 3) - M^2(N_C \to \infty)}{M^2(N_C = 3)}\left(\frac{3}{N_C} - 1\right)\right]$$

Scenario 4: D-waves are included
The couplings are proportional to $\sqrt{N_C}$


n=0
n=1
n=2
n=3
F_0^{21}
F_1^{21}
F_2^{21}
F_3^{21}
N_C
S-1
S-2
S-3
S-4
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30

Thus, heavier vector resonances are needed. For larger n they have no impact due to their large mass. **S-5 includes a ρ'.**

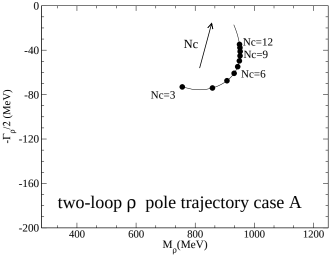


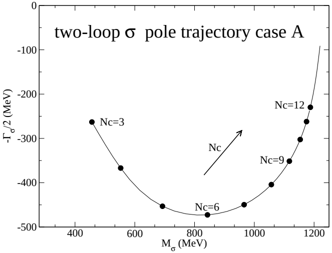
A 40% BR to $\pi\pi$ is needed for this other resonance ρ'

IAM: Leading large N_C behavior

$$L_i(N_c) = L_i(3)\left(\frac{N_C}{3}\right)^{\alpha}$$

Average duality is fulfilled for two-loop IAM but not for one-loop IAM





M_ρ in one-loop IAM at large N_C is $\simeq$ 750 MeV

M_ρ in two-loop IAM at large N_C is $\simeq$ 940 MeV

$q\bar{q}$-like structure for large N_C

$\sqrt{s_\sigma} = 1.2 - i\,0.23$ GeV at $N_C = 12$

For N_C large

$$\mathrm{Im}\,A^{(I=1)}_{J=1}(s) = \frac{\pi M_\rho \Gamma_\rho}{\bar\sigma(M_\rho^2)}\delta(s-M_\rho^2) \quad , \quad \bar\sigma(s)=\sqrt{1-4m_\pi^2/s}$$

$$\int_{\nu_1}^{\nu_{\mathbf{max}}} \nu^{-n}\,\mathrm{Im}\,A^{I=2}_{t,\rho}(\nu,t) = -\frac{3}{2}\frac{\pi\,\Gamma_\rho\,M_\rho^{1-2n}}{\bar\sigma(M_\rho^2)}$$

Ratio between two- and one-loop contributions

$$\frac{\Gamma_{\rho,\mathrm{two-loop}}}{\Gamma_{\rho,\mathrm{one-loop}}}\left(\frac{M_{\rho,\mathrm{two-loop}}}{M_{\rho,\mathrm{one-loop}}}\right)^{1-2n}\frac{\bar\sigma(M^2_{\rho,\mathrm{one-loop}})}{\bar\sigma(M^2_{\rho,\mathrm{two-loop}})}$$

For $n=3$ one has a reduction of 0.43.

$\rho(770)$ contribution is strongly suppressed in two-loop IAM.**Why?**

To which resonance does the $f_0(600)$ pole correspond at large N_C?

If it were a $\bar{q}q$ [I] (think of the ρ) its mass movement will be much smaller

Our study answers these two questions:
First: The short distance constraints imply, when including crossed ρ-exchanges, a transition of G_V from

$$G_V(3) \simeq \frac{F_\pi(3)}{\sqrt{2}} \to G_V(\infty)^{Nor} \simeq \frac{F_\pi(3)}{\sqrt{3}}$$

Second: This $\bar{q}q$ resonance around 1 GeV corresponds to S_1 that contributes for $N_C = 3$ to the $f_0(980)$ resonance pole (another contribution is a $K\bar{K}$ bound state Oset, JAO NPA'97)

Running of pole positions with N_c

For the first time the N_c dependence of the pseudo-Goldstone masses and mixing angle are taken into account for determining resonance properties with increasing N_c.

There is not large N_c limit of $SU(3)$ χPT.

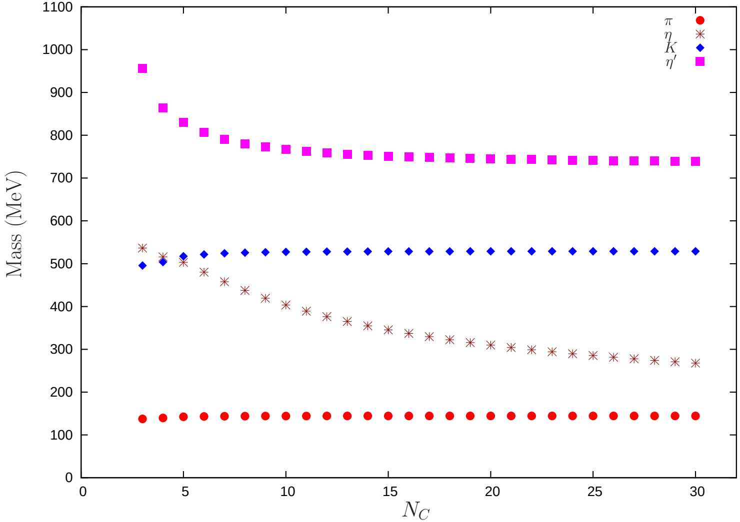
The η_1 becomes the ninth pseudo-Goldstone boson.

$$M_0^2 \sim \Lambda_2 \sim 1/N_c$$
$$c_d \sim c_m \sim \widetilde{c}_d \sim \widetilde{c}_m \sim G_V \sim F \sim \sqrt{N_c}$$
$$M_V^2 \sim M_{S_8}^2 \sim M_{S_1}^2 \sim B \sim a_{SL} \sim \mathcal{O}(N_c^0)$$

We take full Scenario 3 (includes subleading $1/N_C$ corrections)

We solve for M_π^2, M_K^2, M_η^2, $M_{\eta'}^2$ and the LO mixing angle θ as functions of N_c

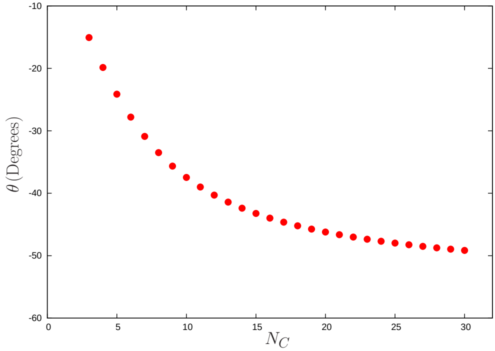
Pseudoscalar masses with varying N_c



Leading order $1/N_c \to \infty$ prediction ($M_0 \to 0$):

$$M_\eta^2 = M_\pi = (139.5^{+4.4}_{-4.6})^2 \text{ MeV}^2 \,,$$
$$M_{\eta'}^2 = 2M_K^2 - M_\pi^2 = (721.5^{+17.4}_{-11.1})^2 \text{ MeV}^2 \,.$$

Ideal Mixing (OZI rule is exact): $\theta = -57.7^\circ$



N

Mas

b

$$\frac{1}{M_V^2 - t} \to \frac{1}{M_V^2} ,$$

NLO local terms in χPT. *Vector Reduced* case (Blue Triangles)

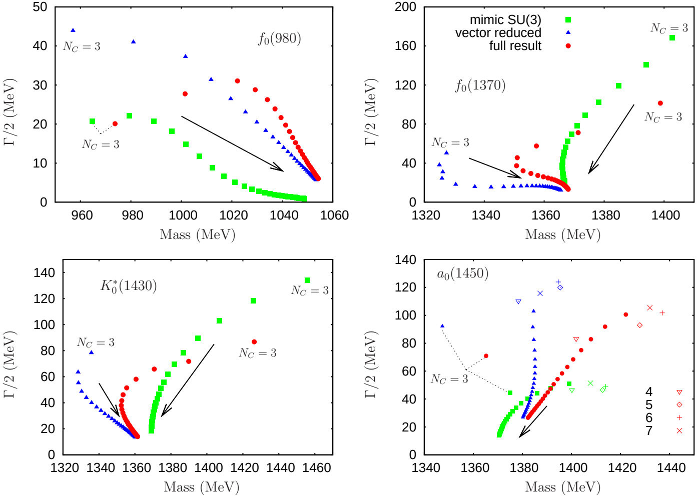
The σ-pole trajectory is then more similar to that of the IAM one-loop study Peláez PRL'04
Sensitivity to higher order local terms.

When full vector propagators are kept their crossed exchange contributions cancel mutually with those from crossed loops along the RHC ($\sqrt{s} \lesssim 1$ GeV) Oller, Oset PRD'99

For increasing N_c loops are further suppressed and this cancellation is spoilt: More sensitivity to the LHC contributions.

The σ pole blows-up in the complex plane (not $q\bar{q}$). Dynamically generated resonance.

Mimic SU(3) case: Mixing is set to zero and η_1 is kept in the loops. η_8, η_1 masses are frozen. Differences highlight the role of η'. Differences for the σ case are negligible.



$f_0(980)$

The Pole moves for $N_c \to \infty$ to the bare scalar singlet on the real axis with mass $M_{S_1} \simeq 1$ GeV.

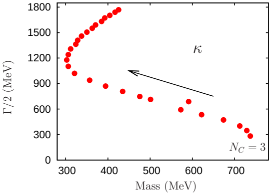
The $f_0(980)$ has an important $K\bar{K}$ bound state contribution to its nature Weinstein, Isgur PRL'82; PRD'83; PRD'90; Weinstein, PRD'93; Janssen *et al.* (Jülich model) PRD'95; Oller, Oset NPA'97; PRD'99.
It disappears in the large N_c limit.

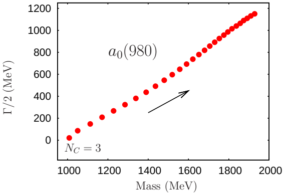
$f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$

All of them evolve to the octet of bare scalar resonances $M_{S_8} = 1370^{+172}_{-57}$ MeV.

Large differences between our full results and mimic SU(3) case, indicating large couplings to η' channels.

κ: Track of $(+,-,+)$ pole





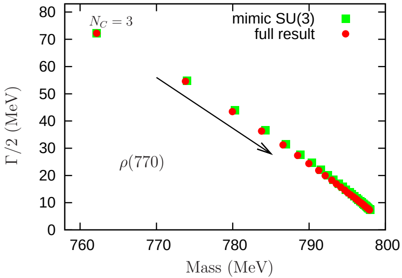
Similar trajectory as for the σ. The mass decreases with N_c and the width increases very fast.

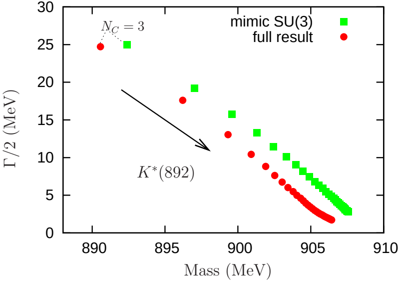
In the vector reduced case the mass keeps increasing with N_c.

$a_0(980)$

It disappears deep in the complex plane, not $q\bar{q}$. Dynamically generated resonance.

$\rho(770)$, $K^*(892)$





$q\bar{q}$ trajectories: Mass $\mathcal{O}(N_c^0)$ and Width $\mathcal{O}(1/N_c)$ Peláez PRL'04

Above $N_c = 13$ the $K\eta$ threshold becomes lighter than the $K^*(892)$ mass (kink in the residue to $K\eta$ at $N_c = 14$).
For the $\phi(1020)$ the situation is the same. There is sensitivity to the slight movement of the nearby $K\bar{K}$ threshold with N_c.

Conclusions

- A complete one loop calculation of all meson-meson scattering amplitudes within $U(3)$ χPT was worked out.
- It is included the explicit exchange of tree level scalar and vector resonances in the s-,t- and u-channels.
- The perturbative amplitudes are unitarized using an approach based on the N/D method, while treating perturbatively the crossed channel dynamics.
- More resonances are generated than included. Dynamical generation of resonances from the self-interactions between the lightest pseudoscalars.

- N_c dependence considered for various resonance quantities, such as pole positions and residues.
- Peculiar trajectories indicating dynamical generation of the lightest scalar resonances σ, κ, $a_0(980)$.
- Bare singlet scalar pole S_1 present in the $f_0(980)$ with mass ~ 1 GeV.
- It does not evolve from the σ and it stands both at $N_c = 3$ and $N_c \to \infty$.
- $f_0(1370)$, $K_0^*(1430)$ and $a_0(1450)$ move to the bare octet of scalar resonances S_8 with mass ~ 1.37 GeV.
- Vector resonances $\rho(770)$, $K^*(892)$ and $\phi(1020)$ follow a textbook $q\bar{q}$ large N_c trajectory.

- Non-trivial QCD constraints are satisfied quite satisfactorily by our partial waves
- They imply interesting relations between the scalar spectrum and the pseudoscalar one and also with the vector resonance spectrum
- Interesting transition between $N_C = 3$ to $N_C \to \infty$. The strength of the $f_0(600)$ (dynamical-)resonance progressively evolves to the S_1 (bare-)resonance
- For semi-local duality this is accompanied by a weakening of the ρ contribution ($G_V \to F_\pi/\sqrt{3}$ for large N_C). This also sheds light to large N_C QCD.
- For the PP correlators this is accompanied by the change of the couplings to the pseudoscalar sources

- Our study of all these facets simultaneoulsy: chiral symmetry, unitarity, analyticity, QCD second spectral function sum rules, QCD semi-local duality and large N_C QCD strengthens our understanding of the scalar spectrum and its nature.
- $f_0(600)$ non-standard $\bar{q}q$ resonance.
- Existence of a bare isosinglet scalar at 1 GeV.
- First $\bar{q}q$ octet at around 1.4 GeV.