

Scalar sectors in multi-Higgs-doublet models

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Outline

- 1 Introduction
- 2 Scalar sector in general 2HDM
- 3 Scalar sector in general NHDM
- 4 Conclusions

EWSB in Standard model

The SM Higgs mechanism of EWSB uses **one doublet** of complex scalar fields:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

which interacts with gauge and matter fields and self-interacts with potential

$$V = -\frac{1}{2}m^2(\phi^\dagger\phi) + \frac{1}{2}\lambda(\phi^\dagger\phi)^2.$$

For positive m^2 it acquires a non zero v.e.v.

$$\langle\phi\rangle \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.$$

Three of four initially massless gauge bosons (W^\pm , Z) acquire masses, while the photon remains massless. The symmetry is broken from $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$.

Multi-Higgs-doublet models

Motivations for EWSB models with several scalar doublets:

- **2HDM** is a simple (yet rich) bSM extension of the Higgs mechanism and is used in MSSM.
- Pursuing the idea of generations in the scalar sector further — ***N*-Higgs-doublet model** (NHDM).
- Many specific variants of NHDM for $N \geq 3$ were suggested (Weinberg's 3HDM, Adler's 4HDM, SUSY version of ν 2HDM, private Higgs model, etc).
- The focus has been on the fermion mass matrices. However, there is **rich physics in the scalar sector** barely touched. Very little is known about what in principle can happen in the scalar sector of NHDM.

Scalar sector in 2HDM

We introduce two electroweak doublets, ϕ_1 and ϕ_2 , and write the scalar potential $V = V_2 + V_4$:

$$V_2 = -\frac{1}{2} \left[m_{11}^2 (\phi_1^\dagger \phi_1) + m_{22}^2 (\phi_2^\dagger \phi_2) + m_{12}^2 (\phi_1^\dagger \phi_2) + m_{12}^{2*} (\phi_2^\dagger \phi_1) \right] ;$$

$$V_4 = \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\ + \frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_2)^2 + \left[\lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2) \right] (\phi_1^\dagger \phi_2) + \text{h.c.}$$

It contains **14 free parameters**: 4 free parameters m_{ab}^2 and 10 λ 's.

Scalar sector in NHDM

We introduce ϕ_a , $a = 1, \dots, N$, and construct the general scalar potential from $(\phi_a^\dagger \phi_b)$'s:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d),$$

with N^2 independent components in Y and $N^2(N^2 + 1)/2$ independent components in Z (e.g. 54 free parameters for $N = 3$).

Digression: how do we study a model?

Typical situation in physics bSM: we introduce a model which contains **free parameters**, which are not initially constrained.

What do we want to find ultimately:

- **Derive phenomenology**: at each point in the parameter space: vacuum, masses, decays, contributions to processes at colliders, etc. — and, eventually, test the model experimentally.
- Get the **qualitative feeling of the model** by constructing the phase diagram in the entire parameter space.

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Are we there with 2HDM?

2HDM has been studied since almost 40 years in hundreds of publications. Computationally, it has been adapted to phenomenological needs (e.g. implemented in MadGraph).

But do we understand this model, at least at the tree-level?

Yes, we are close to it, but only thanks to novel approaches proposed in the last few years.

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Consider the most general **quartic part** of the potential:

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Does it have any additional symmetry (beyond the EW invariance) for a **generic set of λ 's**?

Yes. Its symmetry group is **at least** $(\mathbb{Z}_2)^3$.

I cannot imagine how one can see this with direct calculations. With the geometric approach to be explained below, it's almost trivial.

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Example 2

CP-violation in the scalar sector of 2HDM.

- Early works: need complex free parameters (e.g. λ_5 or m_{12}^2).
- Is it sufficient? **No**, as there are models with complex parameters and no *CP*-violation (examples were found by trial and error).
- What are then the necessary and sufficient conditions for *CP*-violation in the scalar sector? As of 2005, controversy in literature existed.
- It turned out later that [*Gunion, Haber (2005)*] were correct, but they used automated Mathematica search among millions(!) of invariants and did not have the full proof → **does not count as understanding!**
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Understanding the structure in the space of models with N doublets is an activity **complementary** to the detailed collider or astroparticle calculations.

It works with the **most general form of the NHDM potential** and gives you the knowledge which you cannot get by focusing just on simple versions of the model.

Again, the most general NHDM potential:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d),$$

We first need to **minimize** it (\rightarrow vacuum) and then expand it at the point of global minimum (\rightarrow spectrum and interactions of the physics Higgs bosons).

There are three kinds of minima:

- **Electroweak vacuum:** $\langle \phi_a \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ for all a .
- **Neutral vacuum** $\langle \phi_a \rangle = \begin{pmatrix} 0 \\ v_a \end{pmatrix}$, with possibly complex v_a .
- **Charge-breaking vacuum:** $\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$, $\langle \phi_a \rangle = \begin{pmatrix} u_a \\ v_a \end{pmatrix}$, some $u_a \neq 0$.

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The **main problem in NHDM**: the general potential **cannot be minimized explicitly** (coupled algebraic equations of high order) → other methods are needed to learn at least something about the general NHDM.

For **2HDM** several basis-invariant techniques were developed:

- **tensorial formalism** [*Botella, Silva (1995); Haber, Gunion, et al (2005-2006)*]
- **geometric approach** [*Nachtmann et al; Ivanov; Nishi; (2004-2008)*].

Many non-trivial results (number and coexistence of minima, symmetries and their violation, phase diagram) were obtained without **explicit minimization of the potential**.

Generalization of this approach to N -doublets leads to more non-trivial mathematics.

General 2HDM

The Higgs potential of general 2HDM depends on fields via four gauge-invariant bilinear combinations:

$$(\phi_1^\dagger \phi_1), (\phi_2^\dagger \phi_2), (\phi_1^\dagger \phi_2), (\phi_2^\dagger \phi_1).$$

Let us construct a singlet and a triplet:

$$r_0 = (\phi_a^\dagger \phi_a) \equiv (\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2),$$
$$r_i = (\phi_a^\dagger \sigma_{ab}^i \phi_b) \equiv \begin{pmatrix} 2\text{Re}(\phi_1^\dagger \phi_2) \\ 2\text{Im}(\phi_1^\dagger \phi_2) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix}.$$

Then, each point (r_0, r_i) describes a unique EW orbit \rightarrow orbit space.

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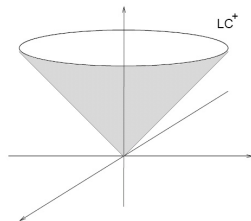
Orbit space

The shape of the orbit space: $r_0 > 0$ and

$$r_\mu r^\mu \equiv r_0^2 - r_i^2 = 4 \left[(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right] \geq 0.$$

The orbit space is the surface and interior of the **forward lightcone** LC^+ .

- **vertex** → EW-symmetric vacuum
- **surface** → neutral vacuum
- **interior** → charge-breaking vacuum



Reparametrization transformation

The key property of the generic potential: any linear (not necessarily unitary!) transformation between ϕ_1 and ϕ_2 gives again the generic potential with the same observables, but with reparametrized coefficients m_{ij}^2 and λ_i .

But if ϕ_a are transformed by $A \in SL(2, C)$, then r_0 and r_i transform under $SO(1, 3)$ as a single 4-vector $r^\mu = (r_0, r_i)$.

- unitary $A \rightarrow$ 3D rotations of r_i ;
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The general Higgs potential

$$\begin{aligned}
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 & + \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\
 & + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} \left[\lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2 \right] \\
 & + \left\{ \left[\lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2) \right] (\phi_1^\dagger \phi_2) + \text{h.c.} \right\}
 \end{aligned}$$

becomes...

... just a **quadratic form** in the orbit space:

$$V = -M_\mu r^\mu + \frac{1}{2} \Lambda_{\mu\nu} r^\mu r^\nu .$$

The **kinetic term** in the lagrangian must be also treated in the reparametrization invariant way:

$$K = K_\mu \rho^\mu, \quad \rho^\mu \equiv (D_\alpha \Phi)^\dagger \sigma^\mu (D^\alpha \Phi).$$

All properties of the most general 2HDM come from the **relative "orientation"** of $\Lambda_{\mu\nu}$, M_μ , K_μ .

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Positivity constraints

$V_4 > 0$ for all non-zero $\phi_i \rightarrow \Lambda_{\mu\nu} r^\mu r^\nu > 0$ on and inside LC^+ . This holds, iff $\Lambda_{\mu\nu}$ can be diagonalized by an $SO(1,3)$ transformation and after diagonalization it takes form

$$\begin{pmatrix} \Lambda_0 & 0 & 0 & 0 \\ 0 & -\Lambda_1 & 0 & 0 \\ 0 & 0 & -\Lambda_2 & 0 \\ 0 & 0 & 0 & -\Lambda_3 \end{pmatrix} \quad \text{with} \quad \Lambda_0 > 0 \text{ and } \Lambda_0 > \Lambda_1, \Lambda_2, \Lambda_3.$$

We do not need to know **explicit expressions** of Λ_i in terms of original λ 's of the potential!

Symmetries

Symmetries of the quartic part ($\Lambda_{\mu\nu} r^\mu r^\nu$) can be easily understood in the $\Lambda_{\mu\nu}$ -diagonal frame.

- if all Λ_i are distinct, the only symmetries are reflections of the three eigenaxes $\rightarrow (\mathbb{Z}_2)^3$ group;
- if two Λ_i coincide, the group is $O(2) \times \mathbb{Z}_2$;
- if all three Λ_i coincide, the group is $O(3)$.

For the entire scalar lagrangian, the symmetry group can be

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Geometric look at minimization of the Higgs potential

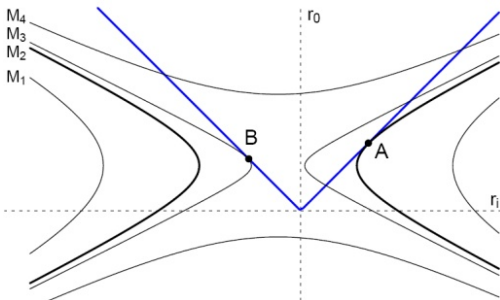
Consider $V(r) = -M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu / 2$ not on the LC^+ , but in the **entire** r^μ space.

Define an **equipotential surface** M_C as all r^μ that give $V(r) = C$.

- Each equipotential surface is an 3D **quadric** (hyperboloid, ellipsoid, etc.) embedded in the 4D space r^μ .
- These equipotential surfaces do not intersect, are ordered, and cover the entire space r^μ .

Geometric look at minimization of the Higgs potential

Minimization of the potential: the unique equipotential surface that **only touches** LC^+ gives the global minimum.



M_1 does not intersect LC^+ . M_2 touches LC^+ at a single point A (global minimum). M_3 touches LC^+ at point B (local minimum).

Phase diagram of the scalar sector in 2HDM

The scalar potential is defined by M_μ and $\Lambda_{\mu\nu}$.

You can always switch to the $\Lambda_{\mu\nu}$ -diagonal basis and find its eigenvalues Λ_0 and Λ_i . Working in this basis, define

$$m_i = \frac{1}{M_0}(M_1, M_2, M_3).$$

Now the phase diagram can be presented in the 3D \vec{m} -space.

There are **two main possibilities**:

- 1 all $\Lambda_i < 0$, $i = 1, 2, 3$.
- 2 some of $\Lambda_i > 0$; identify the largest among them (for example, Λ_2).

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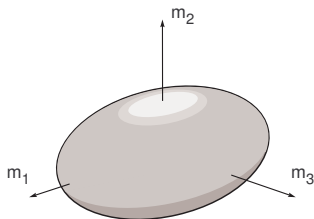
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Case 1

Consider the ellipsoid:

$$\frac{m_1^2}{a_1^2} + \frac{m_2^2}{a_2^2} + \frac{m_3^2}{a_3^2} = 1, \quad \text{where} \quad a_i = \frac{|\Lambda_i|}{\Lambda_0}.$$

- If \vec{m} lies inside, the global minimum is **charge breaking**;
- If \vec{m} lies outside, the global minimum is **neutral** and respects all additional symmetries of the potential.

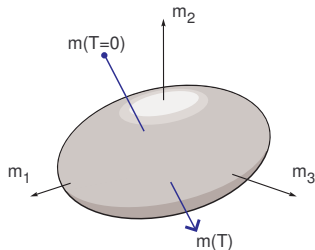


Case 1

If \vec{m} moves in this space (for example, due to finite temperature in the early Universe) and goes through this ellipsoid, one gets **charge-breaking** and **charge-restoring** phase transitions.

At the tree-level, these are second-order phase transitions.

It would be very interesting to see if this scenario of thermal evolution of early Universe has any characteristic **cosmological signatures**.

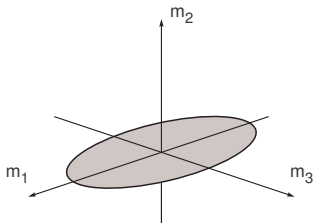


Case 2

Assuming $\Lambda_2 > 0, \Lambda_1, \Lambda_3$, consider the ellipse lying in the (m_1, m_3) -plane:

$$\frac{m_1^2}{b_1^2} + \frac{m_3^2}{b_3^2} = 1, \quad \text{with} \quad b_i = \frac{\Lambda_2 - \Lambda_i}{\Lambda_0 - \Lambda_2}.$$

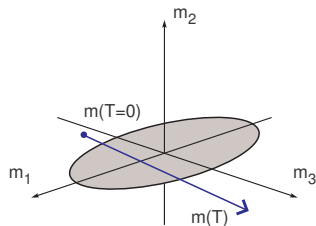
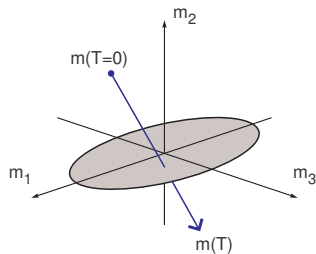
- If \vec{m} lies in the plane and inside the ellipse, there are two global minima with **spontaneous CP-violation**.
- If \vec{m} lies in the plane and outside the ellipse, there is a single global minimum which is **CP-conserving**.
- If \vec{m} lies out of the plane, then **CP is broken explicitly**. Still, if it lies slightly above or below the ellipse, there are **two minima** at different depths.



Case 2

Again, if \vec{m} moves in this phase diagram, several phase transitions are possible.

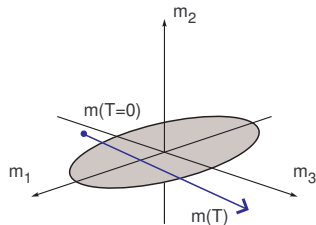
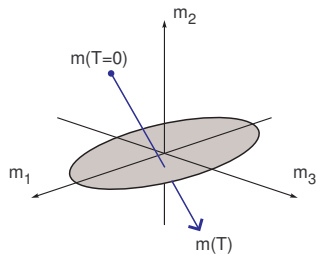
- If \vec{m} punctures the ellipse along its out-of-plane trajectory, then a **first order phase transition** (at the tree level!) takes place.
- If \vec{m} lies in the plane and crosses the ellipse, then second order **spontaneous CP-breaking and CP-restoring** phase transitions take place.
- If \vec{m} approaches the rim of the ellipse, the model exhibits yet **another kind of critical behavior** (with different critical exponents!) than above.



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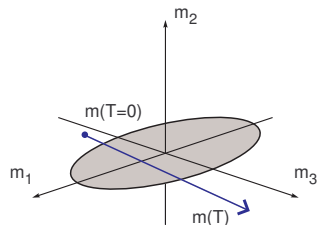
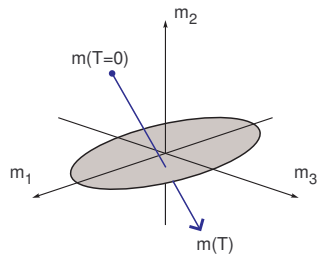
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Orbit space in NHDM

We introduce $r^\mu = (r_0, r_i)$, $i = 1, \dots, N^2 - 1$, via

$$r_0 = \sqrt{\frac{2}{N(N-1)}}(\phi_a^\dagger \phi_a), \quad r_i = \phi_a^\dagger \lambda_{ab}^i \phi_b.$$

In 2HDM the orbit space was just the surface and the interior of the 4D-cone ($r_0 > 0$, $r_0^2 - r_i^2 \geq 0$). In NHDM the orbit space is a **complicated shape** inside an N^2 -dimensional cone.

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A big motivation to study several Higgs doublets is that **symmetries** can be encoded in the Higgs potential.

- Which groups can be realized as symmetry groups of the NHDM Higgs potential?
- How can these groups spontaneously break after EWSB?
- How to find the symmetry group of a given potential?

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Realizable symmetry groups

Definition:

we call a symmetry group G **realizable** if there exists a G -symmetric potential which is not symmetric under a larger symmetry group containing G .

Example:

$(\phi_1^\dagger \phi_1)$ is symmetric under cyclic group \mathbb{Z}_p , for any integer p , generated by phase rotations by $2\pi/p$. But each \mathbb{Z}_p is not a realizable symmetry because $(\phi_1^\dagger \phi_1)$ is $U(1)$ -symmetric under arbitrary phase rotation.

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- I skip the details, but one important complication is that they must be subgroups not of $SU(N)$, but of $PSU(N) \simeq SU(N)/\mathbb{Z}_N$.
- For example, consider the following terms in the 4HDM potential:

$$\lambda(\phi_1^\dagger\phi_3)(\phi_1^\dagger\phi_4) + \lambda'(\phi_2^\dagger\phi_1)(\phi_2^\dagger\phi_4) + \lambda''(\phi_3^\dagger\phi_2)(\phi_3^\dagger\phi_4).$$

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In NHDM for $N \geq 3$ a novel class of symmetries exists: symmetries which are present in the potential but are **necessarily broken** in the vacuum [*Ebrahimi-Keus, Ivanov (2011)*].

We termed them **frustrated symmetries**, because the origin of this phenomenon is mathematically similar to frustration in condensed matter physics.

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Conclusions

- Understanding properties of the general NHDM scalar potential is important, but it requires rather complicated mathematical tools.
- We have developed such tools, both algebraic and geometric, and used them to study several aspects of the scalar sector of NHDM (number and coexistence of minima, symmetries and their breaking).
- For 2HDM, the full phase diagram can be constructed in the tree-level approximation.
- For $N > 2$, only partial success was achieved. In particular, we developed a strategy that identifies all realizable abelian symmetry groups for any number of doublets.
- We are continuing the systematic study of what's possible with N doublets and also do more detailed phenomenological analysis of some particular cases.