Reducing the combinatorial uncertainties using kinematic variables

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In collaboration with K. Choi and D. Guadagnoli, arXiv:1109.2201, *submitted to JHEP.*

Combinatorics?

* ... is a branch of mathematics concerning the study of finite or countable discrete structures. Aspects of combinatorics include **counting the structures of a given kind and size**, deciding when certain criteria can be met, and constructing and analyzing objects **meeting the criteria**, finding "largest", "smallest", or "optimal" objects, ...

* A mathematician who studies combinatorics is called a *combinatorialist*.

...Wikipedia, rev. 12 Oct. 2011

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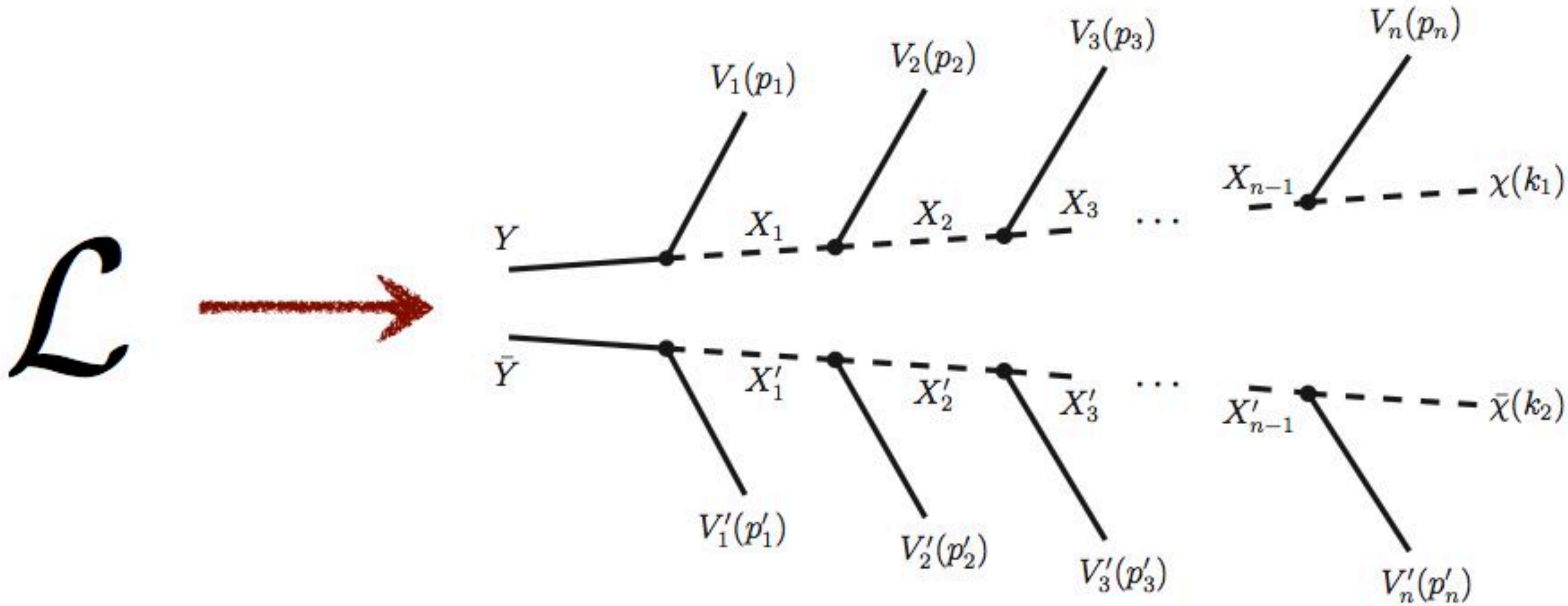
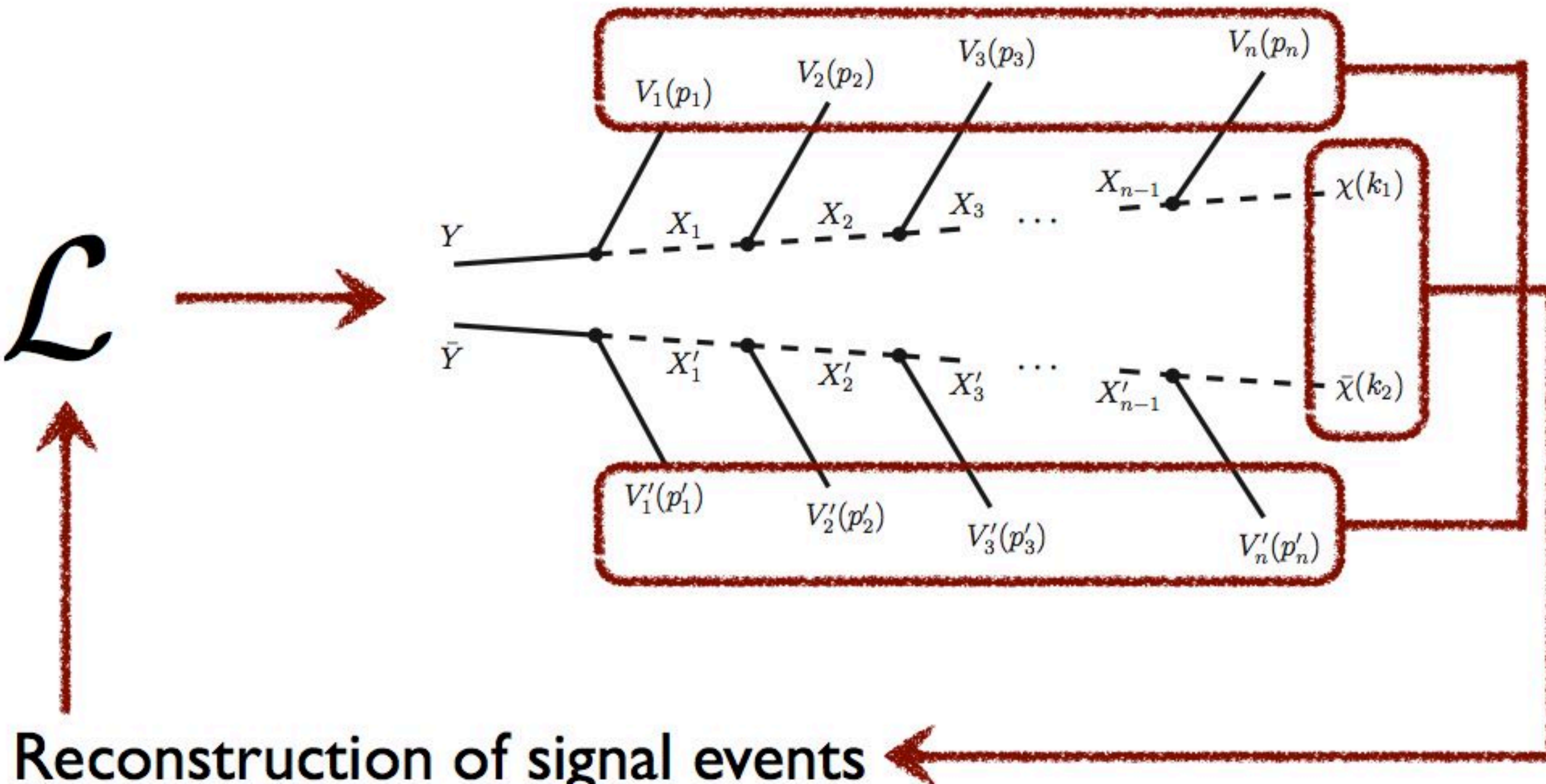
particle physicist studying LHC

Combinatorics in collider signature



Combinatorics in collider signature



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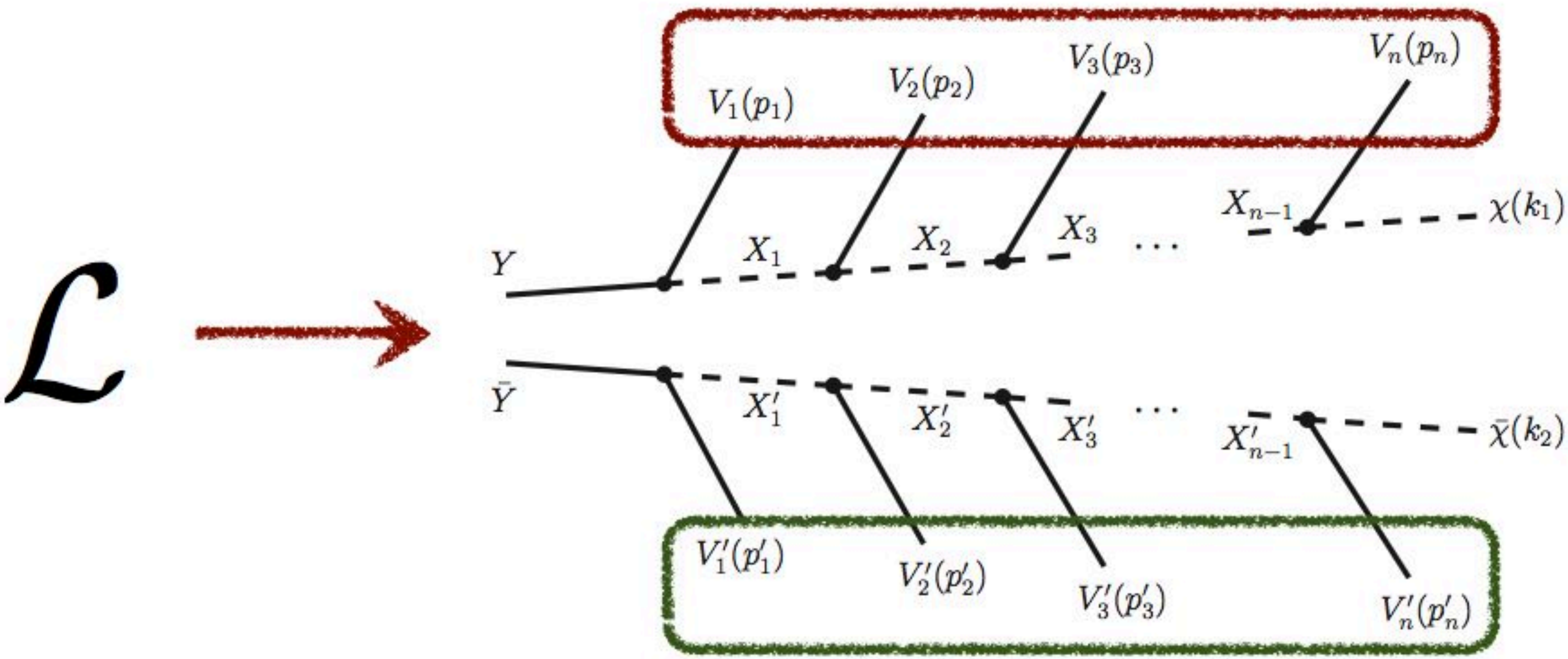
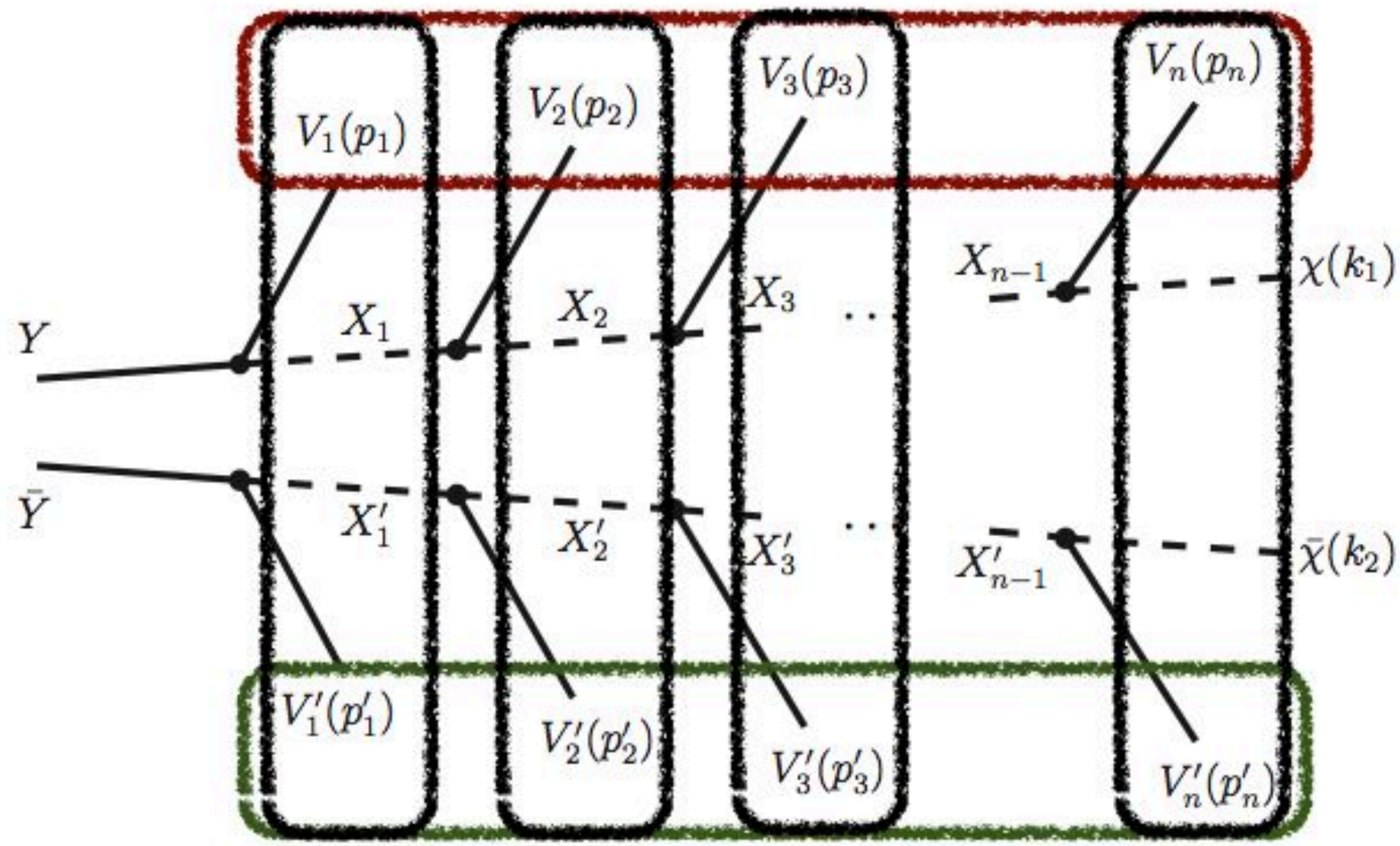


Reconstruction of signal events

* Combinatorial ambiguities of pairing

Combinatorics in collider signature

$\mathcal{L}$ →



Reconstruction of signal events

* Combinatorial ambiguities of pairing and ordering

Combinatorics in collider signature: example (I)

$$Z \to e^+ e^-$$

$$Z \to \mu^+ \mu^-$$

No combinatorial ambiguity

Combinatorics in collider signature: example (I)

$$Z \to e^+ e^-$$

$$Z \to e^+ e^-$$

# of possible pairings = 2

can be resolved by constructing invariant masses:

$|(p^{e^+} + p^{e^-})^2 - m_Z^2| = 0$ for right pairing,

$|(p^{e^+} + p^{e^-})^2 - m_Z^2| \neq 0$ for wrong pairing,

on an event-by-event basis.

Combinatorics in collider signature: example (II)

$$\tilde{q} \to \tilde{\chi}_2^0 q \to l^\pm \tilde{l}^\mp q \to \tilde{\chi}_1^0 l^\pm l^\mp q$$

$$\tilde{q} \to \tilde{\chi}_1^0 q$$

$$(m_{l^\pm l^\mp}^{\max})^2 = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}}^2}$$

Combinatorics in collider signature: example (II)

$$\tilde{q} \to \tilde{\chi}_2^0 q \to l^{\pm}\tilde{l}^{\mp} q \to \tilde{\chi}_1^0 l^{\pm} l^{\mp} q$$

$$\tilde{q} \to \tilde{\chi}_1^0 q$$

main background: 2 jets + 2 leptons + missing ET

$$\tilde{q} \to \tilde{\chi}_1^{\pm} q' \to \tilde{\chi}_1^0 l^{\pm} \nu q'$$

$$\tilde{q} \to \tilde{\chi}_1^{\mp} q' \to \tilde{\chi}_1^0 l^{\mp} \nu q'$$

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same flavor leptons

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$$\tilde{q} \to \tilde{\chi}_1^\mp q' \to \tilde{\chi}_1^0 l^\mp \nu q'$$

no flavor correlation:
(OSSF)=(OSDF)

Flavor subtraction = (OSSF) - (OSDF)

$$= e^+ e^- + \mu^+ \mu^- - e^+ \mu^- - e^- \mu^+$$

Combinatorics in collider signature: example (II)

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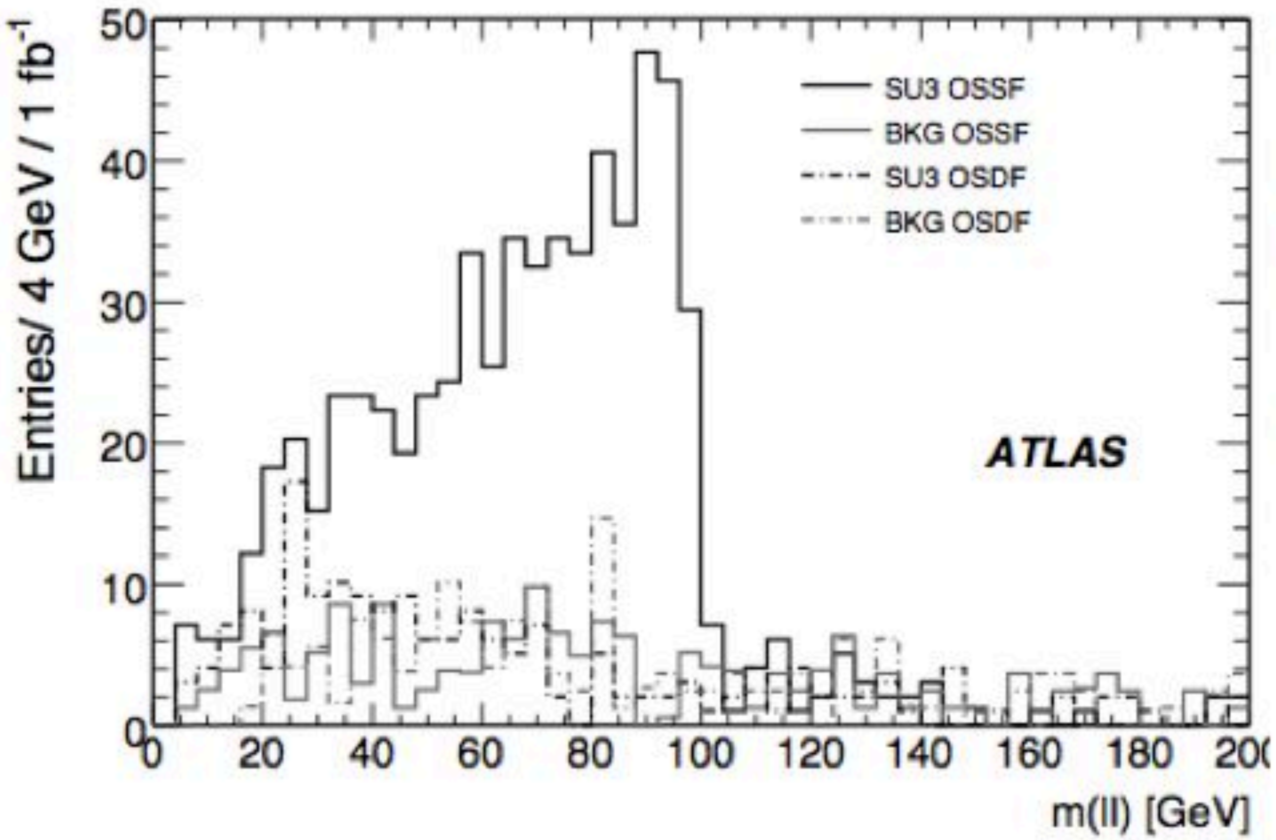

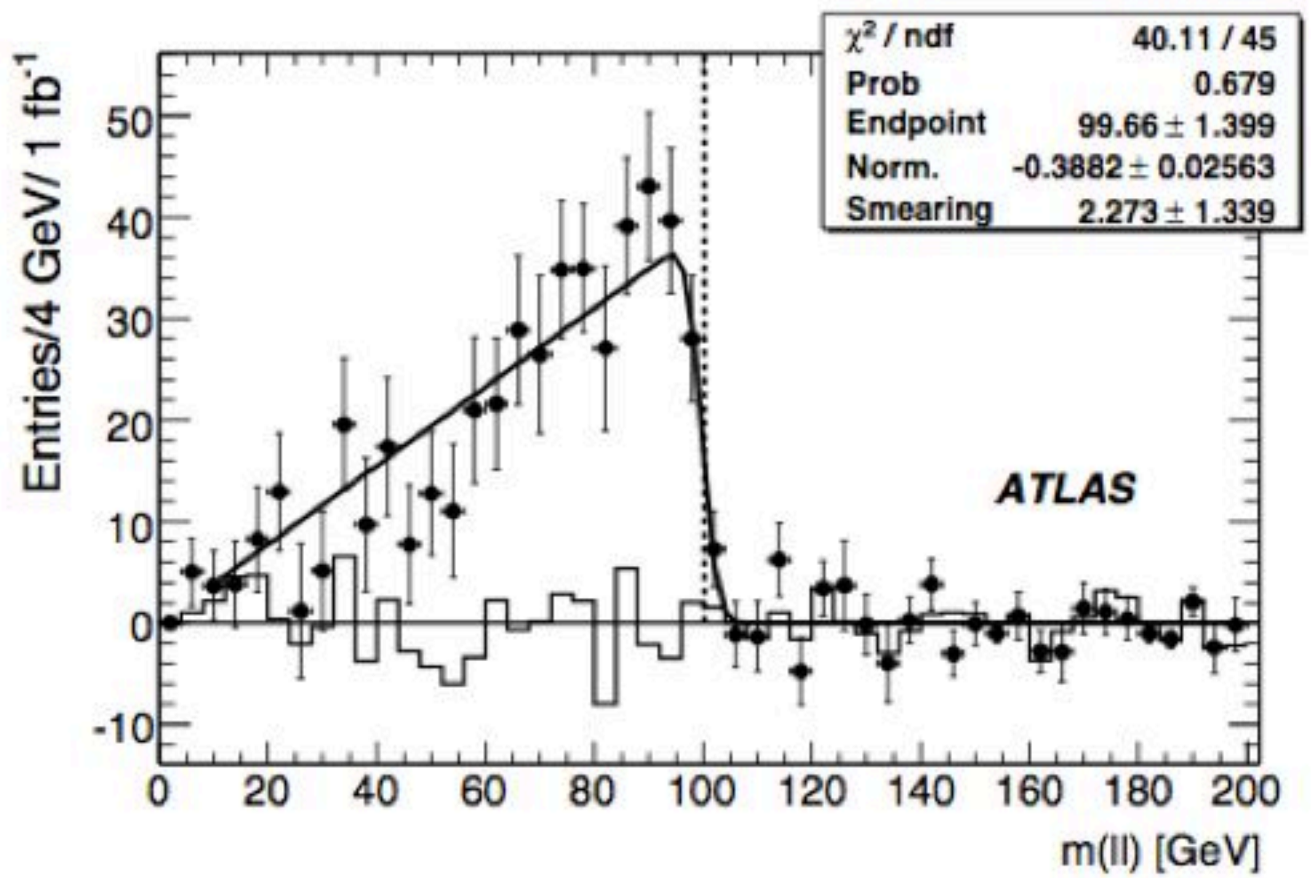

taken from ATLAS TDR (2009)

Flavor subtraction = (OSSF) - (OSDF)

$$= e^+e^- + \mu^+\mu^- - e^+\mu^- - e^-\mu^+$$

Combinatorics in collider signature: top-pair

$$t \to bW^+ \to bl^+\nu$$

$$\bar{t} \to \bar{b}W^- \to \bar{b}l^-\bar{\nu}$$

* Due to the charge ambiguity on b jets,
  # of possible pairings = 2:
  $\{l_1, b_1\}$ & $\{l_2, b_2\}$ for right pairing,
  $\{l_1, b_2\}$ & $\{l_2, b_1\}$ for wrong pairing.

$$t_1 + t_2 \to b_1(p^{b_1})l_1(p^{l_1})\nu_1(k_1) + b_2(p^{b_2})l_2(p^{l_2})\nu_2(k_2)$$

Resolving the combinatorial ambiguity: strategy

* exploit kinematic variables that have predictable features: edge, threshold, peak, ... ➤ ‘test variables’

* take a partition obeying the features as much as possible (wrong partitions will generally not obey because of no correlation).

Resolving the combinatorial ambiguity: strategy

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only depend on the decay topology
➤ 'model-independent'

Combinatorics in collider signature: top-pair

$$(m_{bl}^{(1)})^2 \equiv (p^{b_1} + p^{l_1})^2, \quad (m_{bl}^{(2)})^2 \equiv (p^{b_2} + p^{l_2})^2$$

$$\leq (m_{bl}^{\max})^2 \approx m_t^2 - m_W^2$$

$$(m_{bl}^{(1')})^2 \equiv (p^{b_2} + p^{l_1})^2, \quad (m_{bl}^{(2')})^2 \equiv (p^{b_1} + p^{l_2})^2,$$

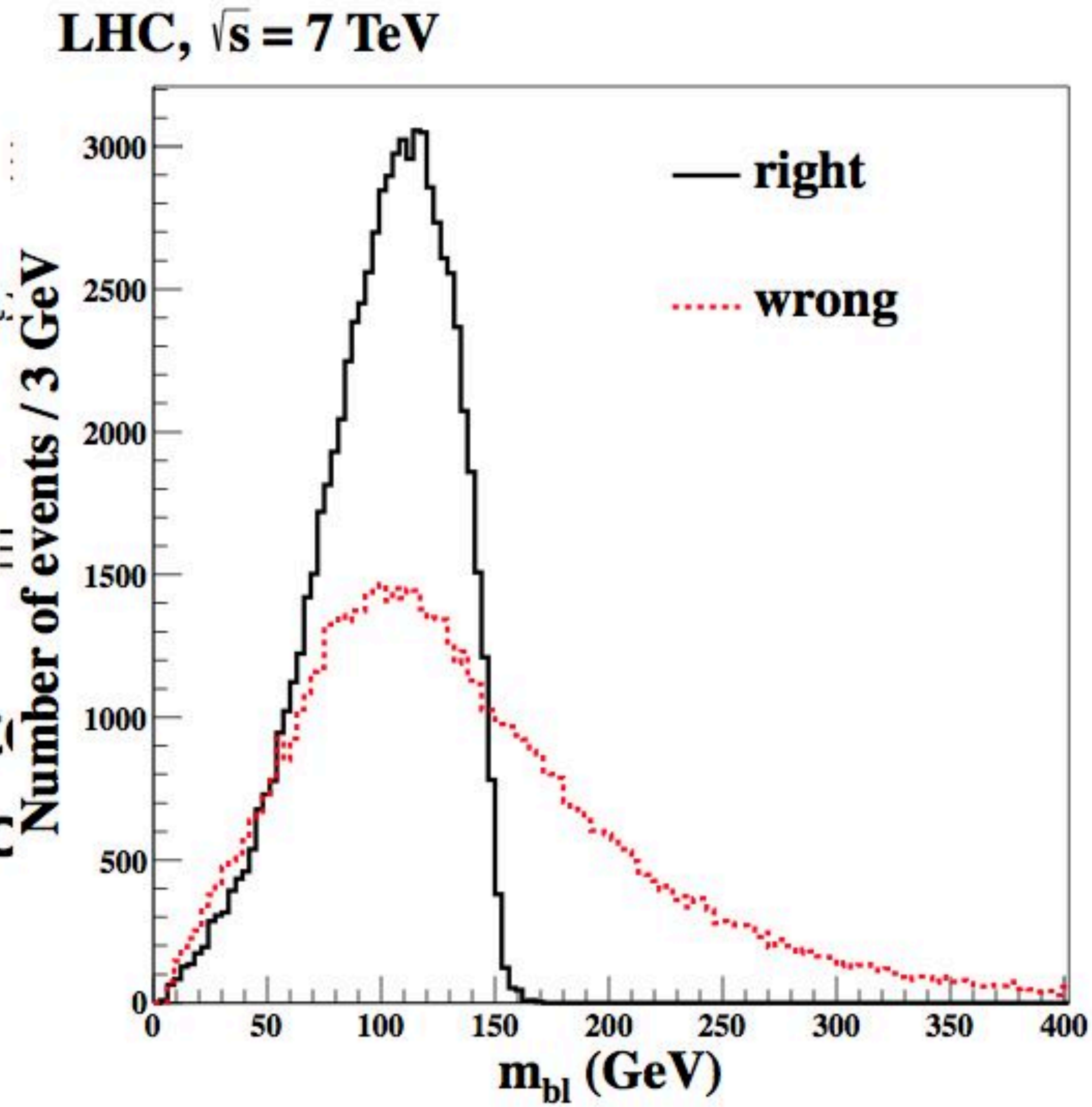
no definite cutoff,
distribution becomes broader for boosted events.

Combinatorics in collider signature: top-pair

$(m_{bl}^{(1)})^2$ ... $+ p^{l_2})^2$

$\leq (m_{bl}^{\mathrm{ma}}$ ...

$(m_{bl}^{(1')})^2$ ... $+ p^{l_2})^2,$



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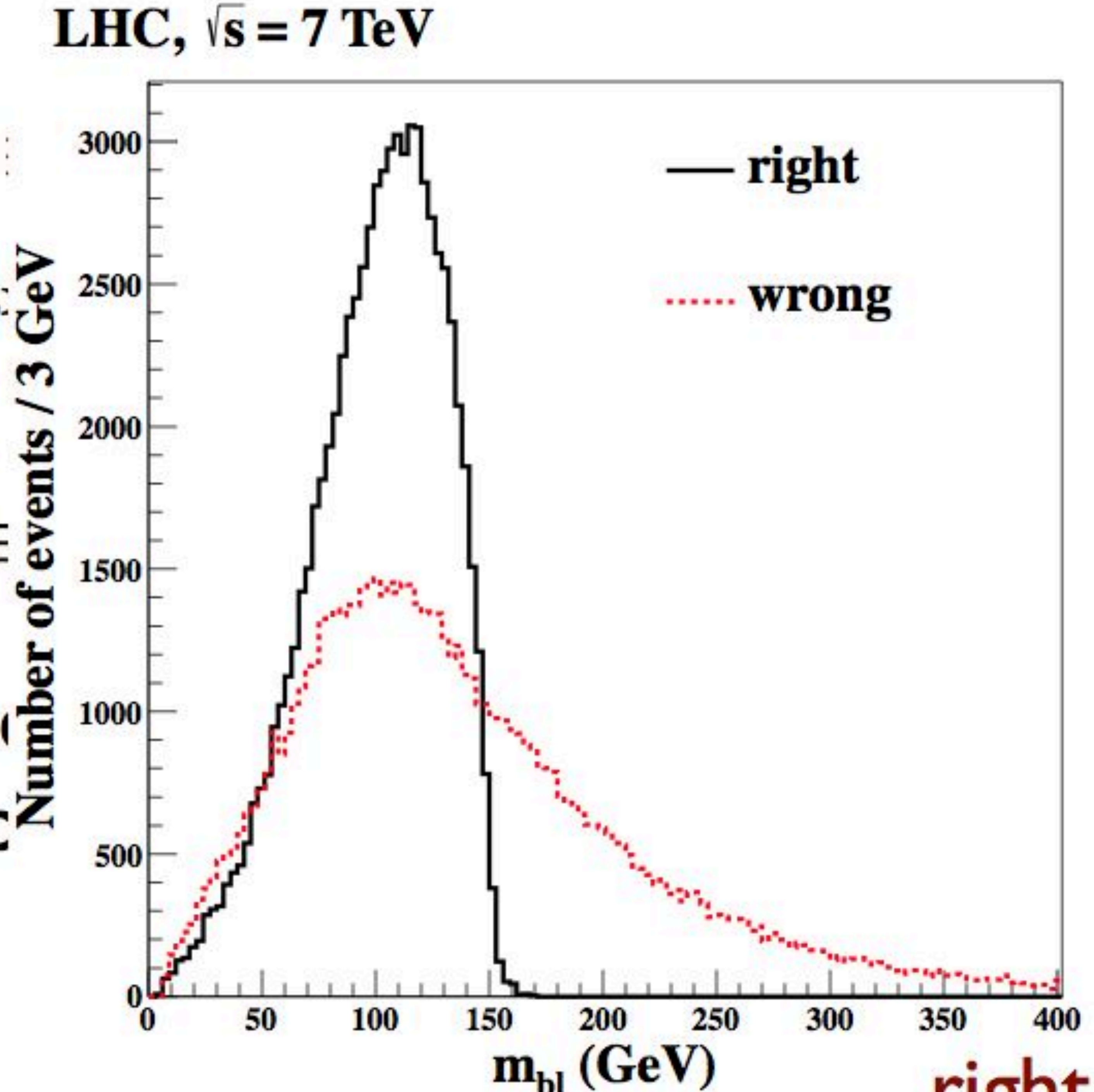
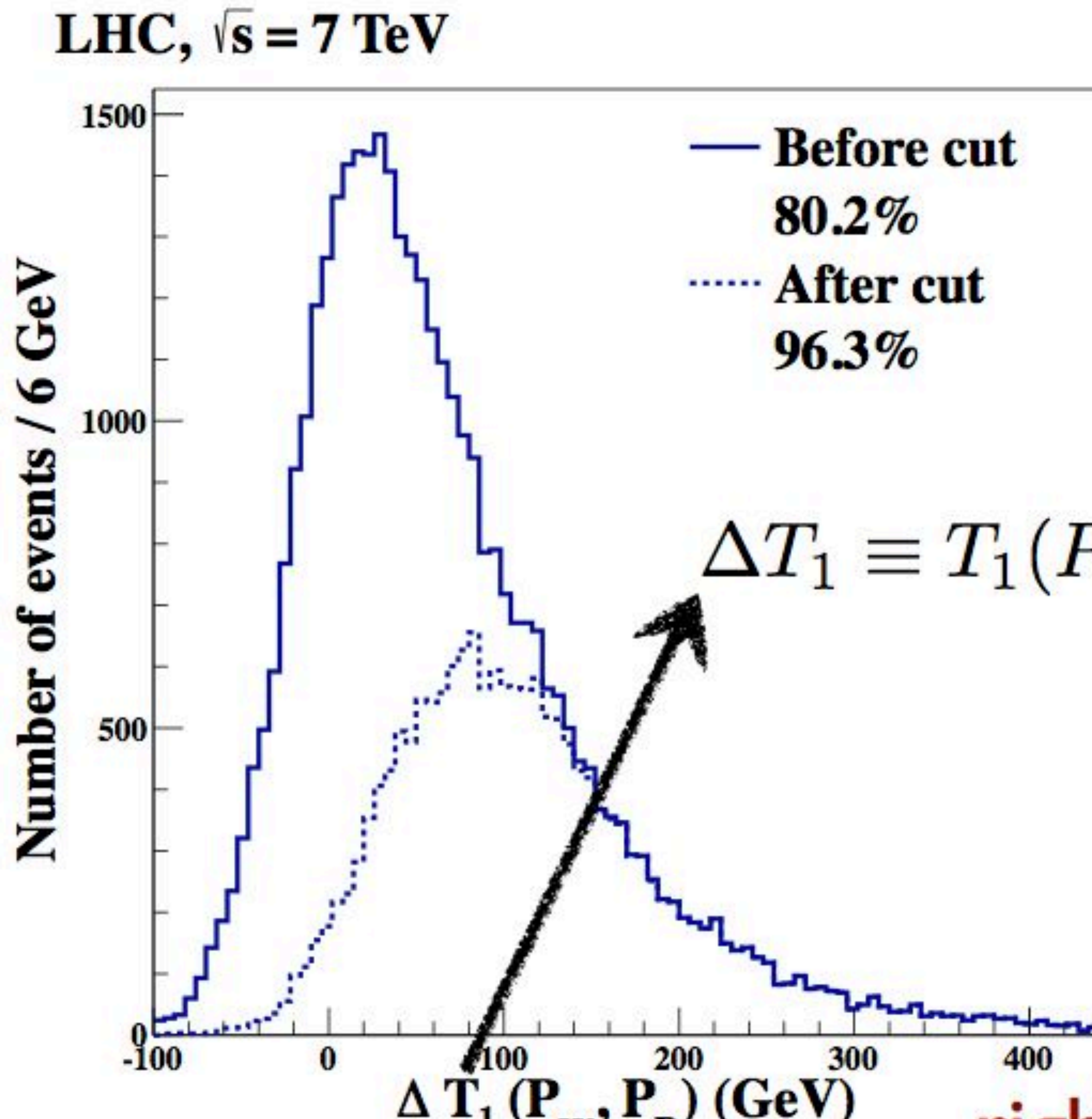


right wrong pairing

$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$ criterion: $T(P_R) < T(P_W)$.

Combinatorics in collider signature: test variable



$$\Delta T_1 \equiv T_1(P_W) - T_1(P_R)$$

right wrong pairing

$$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$$

criterion: $T(P_R) < T(P_W)$.

Combinatorics in collider signature: test variable

C.Lester, D.Summers, PLB(1999)

$$M_{T2} \equiv \min_{\mathbf{k}_{1T}+\mathbf{k}_{2T}=\mathbf{p}_T^{\text{miss}}} \left[\max \left\{ M_T^{(1)},\, M_T^{(2)} \right\} \right]$$

$$M_T^{(i)}(m_{bl}^{(i)},\, \mathbf{p}_T^{b_i} + \mathbf{p}_T^{l_i},\, \mathbf{k}_{iT})$$

: transverse mass in each decay chain

$M_{T2} \leq m_t$ for right paring.

Combinatorics in collider signature: test variable

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: transverse mass in each decay chain

$M_{T2} \leq m_t$ for right paring.

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

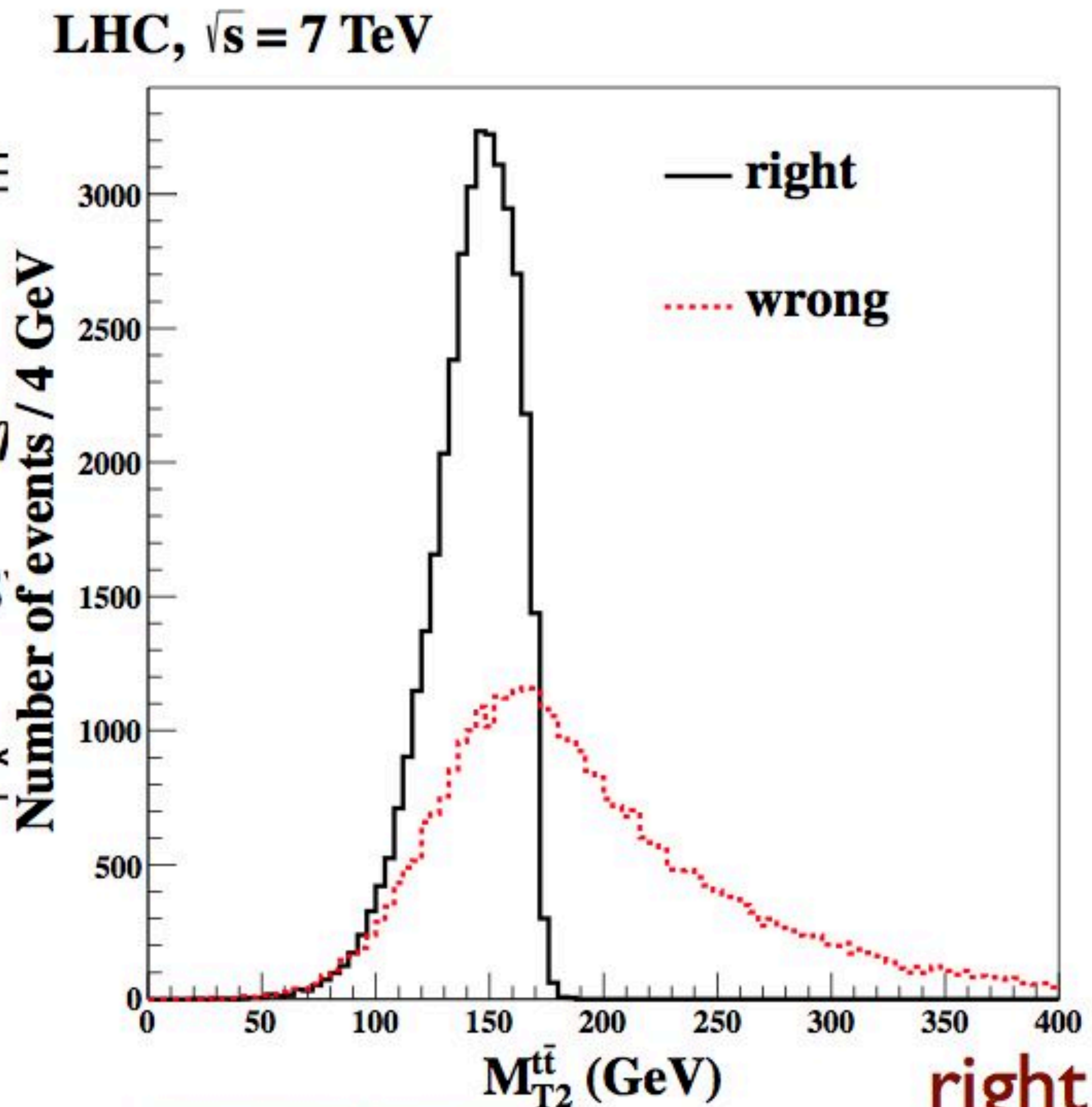
Combinatorics in collider signature: test variable

$M_{T2} \equiv$ ... $M_T^{(2)} \}]$

$M_T^{(i)}($ ...

: trans...

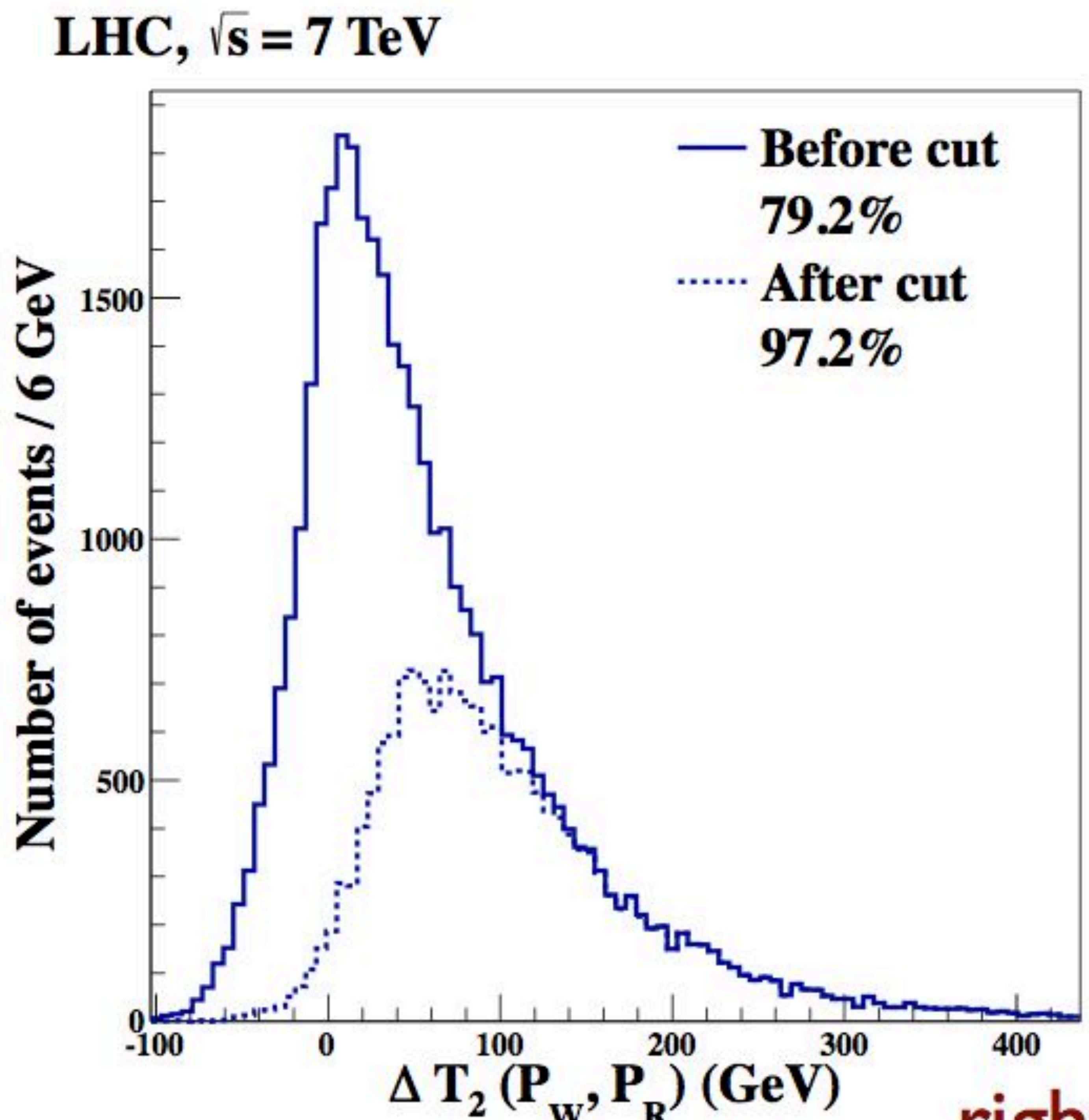
M_{T2} ...



right wrong pairing

$T_2 \equiv M_{T2}^{tt}$ criterion: T(P_R) < T(P_W).

Combinatorics in collider signature: test variable



right wrong pairing

$T_2 \equiv M_{T2}^{tt}$ criterion: $T(P_R) < T(P_W)$.

Combinatorics in collider signature: test variable

$$M_{T2} \equiv \min_{\mathbf{k}_{1T}+\mathbf{k}_{2T}=\mathbf{p}_T^{\rm miss}} \left[\max\left\{ M_T^{(1)},\, M_T^{(2)} \right\} \right]$$

also gives the invisible (neutrino) transverse momenta as a result of minimization.

For given k_T values, k_L can be obtained by on-shell eqs.:

$$(p^b + p^l + k)^2 = m_t^2,$$

$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

Combinatorics in collider signature: test variable

$$M_{T2} \equiv \min_{\mathbf{k}_{1T}+\mathbf{k}_{2T}=\mathbf{p}_T^{\rm miss}} \left[\max\left\{ M_T^{(1)},\, M_T^{(2)} \right\} \right]$$

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momen...

... k_L can be obtained by on-shell

p..:

$$(p^b + p^l + k)^2 = m_t^2,$$

$$(p^l + k)^2 = m_W^2.$$

M_{T2}-assisted on-shell (MAOS) reconstruction

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

W.S.Cho, K.Choi, Y.G.Kim, CBP, PRD(2009)

Combinatorics in collider signature: test variable

$k^{\text{maos}-WW}$ vs $k^{\text{maos}-t\bar{t}}$

$$(p^b + p^l + k)^2 = m_t^2,$$
$$(p^l + k)^2 = m_W^2.$$

top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

Combinatorics in collider signature: test variable

$k^{\mathrm{maos}-WW}$ vs $k^{\mathrm{maos}-t\bar{t}}$

If $k^{\mathrm{maos}-WW} = k^{\mathrm{true}}$

then $(p^b + p^l + k^{\mathrm{maos}-WW})^2 = m_t^2$

If $k^{\mathrm{maos}-t\bar{t}} = k^{\mathrm{true}}$

then $(p^l + k^{\mathrm{maos}-t\bar{t}})^2 = m_W^2$

$$(p^b + p^l + k)^2 = m_t^2,$$

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top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

W-pair subsystem (2l+MET): M_{T2}^{WW}

Combinatorics in collider signature: test variable

$k^{\mathrm{maos}-WW}$

If $k^{\mathrm{maos}-WW} = k^{\mathrm{true}}$

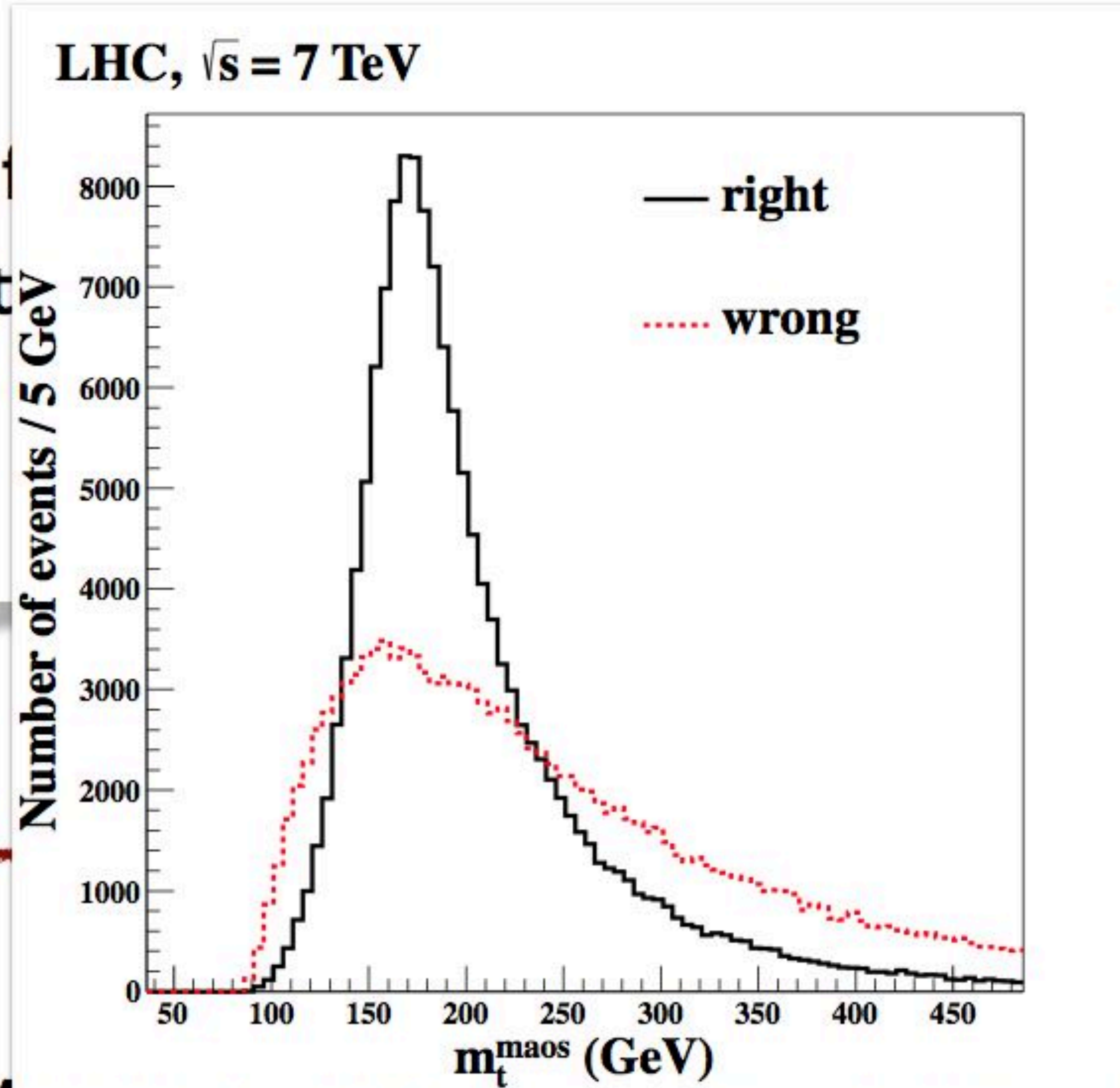
then $(p^b + p^l + k^{\mathrm{maos}-WW})^2 = m_t^2$

$\equiv (m_t^{\mathrm{maos}})^2$

$(p^l + k)^2 = m_W^2.$

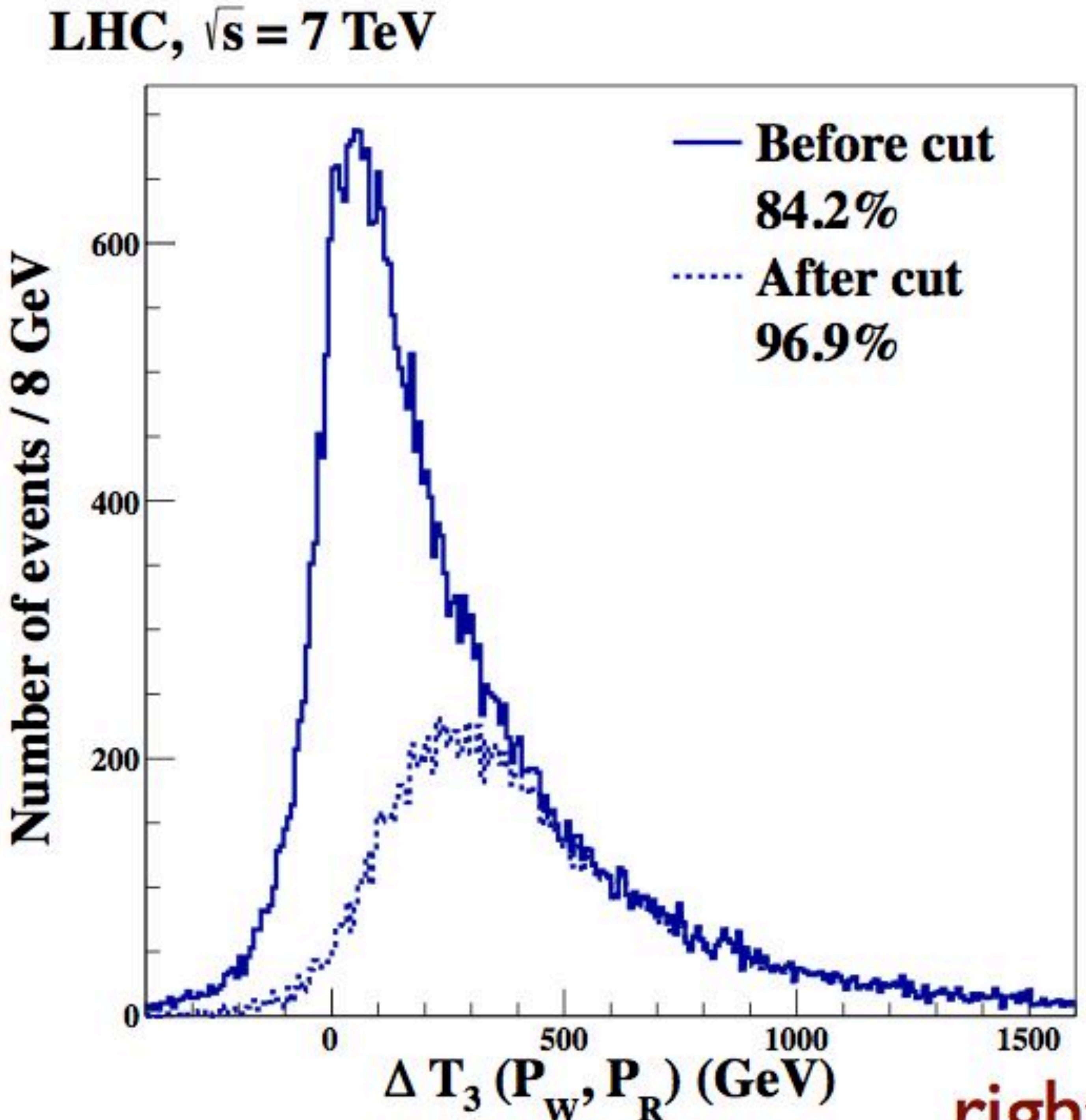
W-pair subsystem (2l+MET): M_{T2}^{WW}

Combinatorics in collider signature: test variable



W-pair subsystem (2l+MET): M_{T2}^{W}

Combinatorics in collider signature: test variable



right wrong pairing

$$T_3 \equiv |m_t^{\mathrm{maos}} - m_t|$$ criterion: $T(P_R) < T(P_W)$.

Combinatorics in collider signature: test variable

$k^{\text{maos}-t\bar{t}}$

If $k^{\text{maos}-t\bar{t}} = k^{\text{true}}$

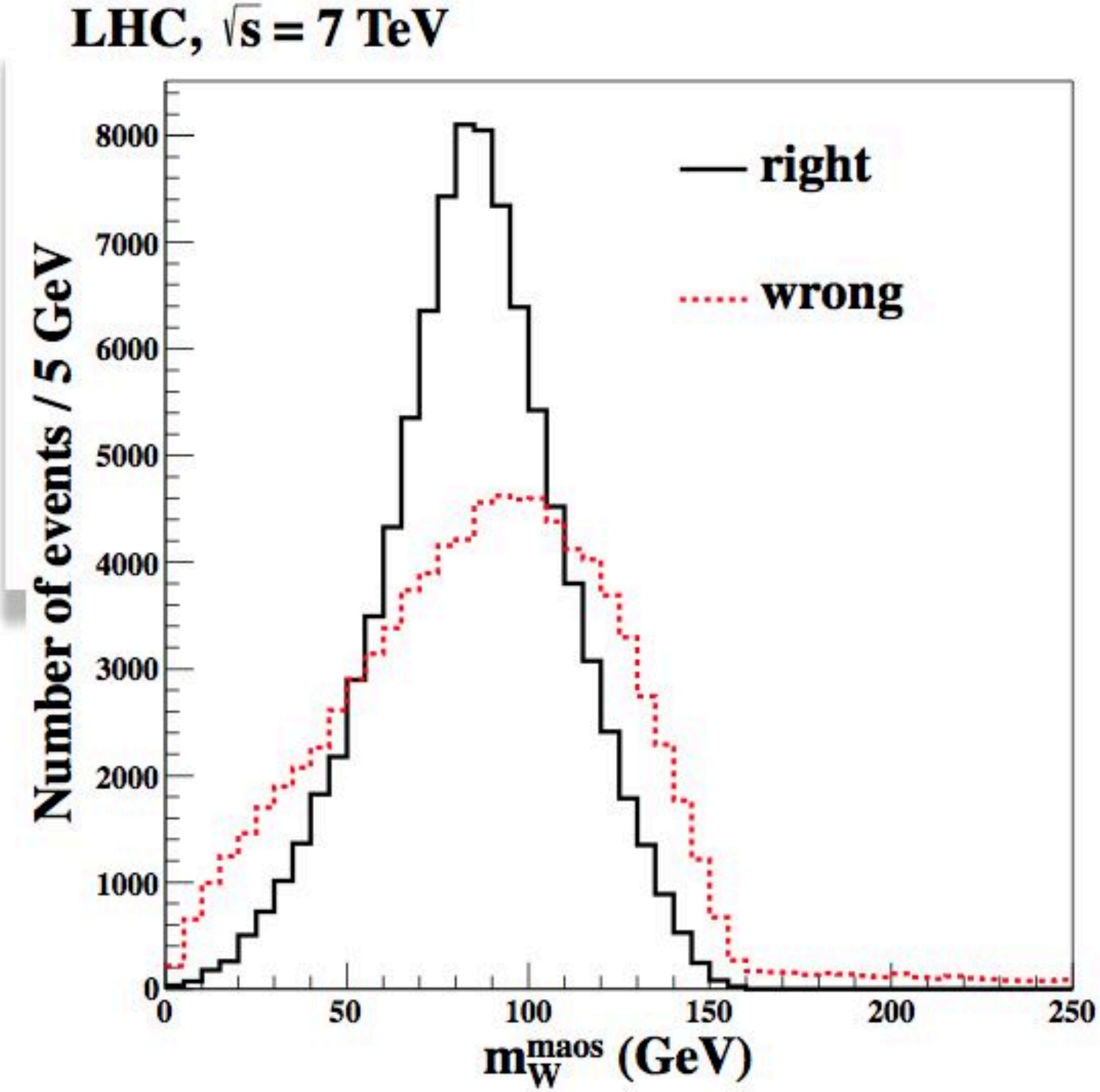
then $(p^l + k^{\text{maos}-t\bar{t}})^2 = m_W^2$

$\equiv (m_W^{\text{maos}})^2$

$$(p^b + p^l + k)^2 = m_t^2,$$

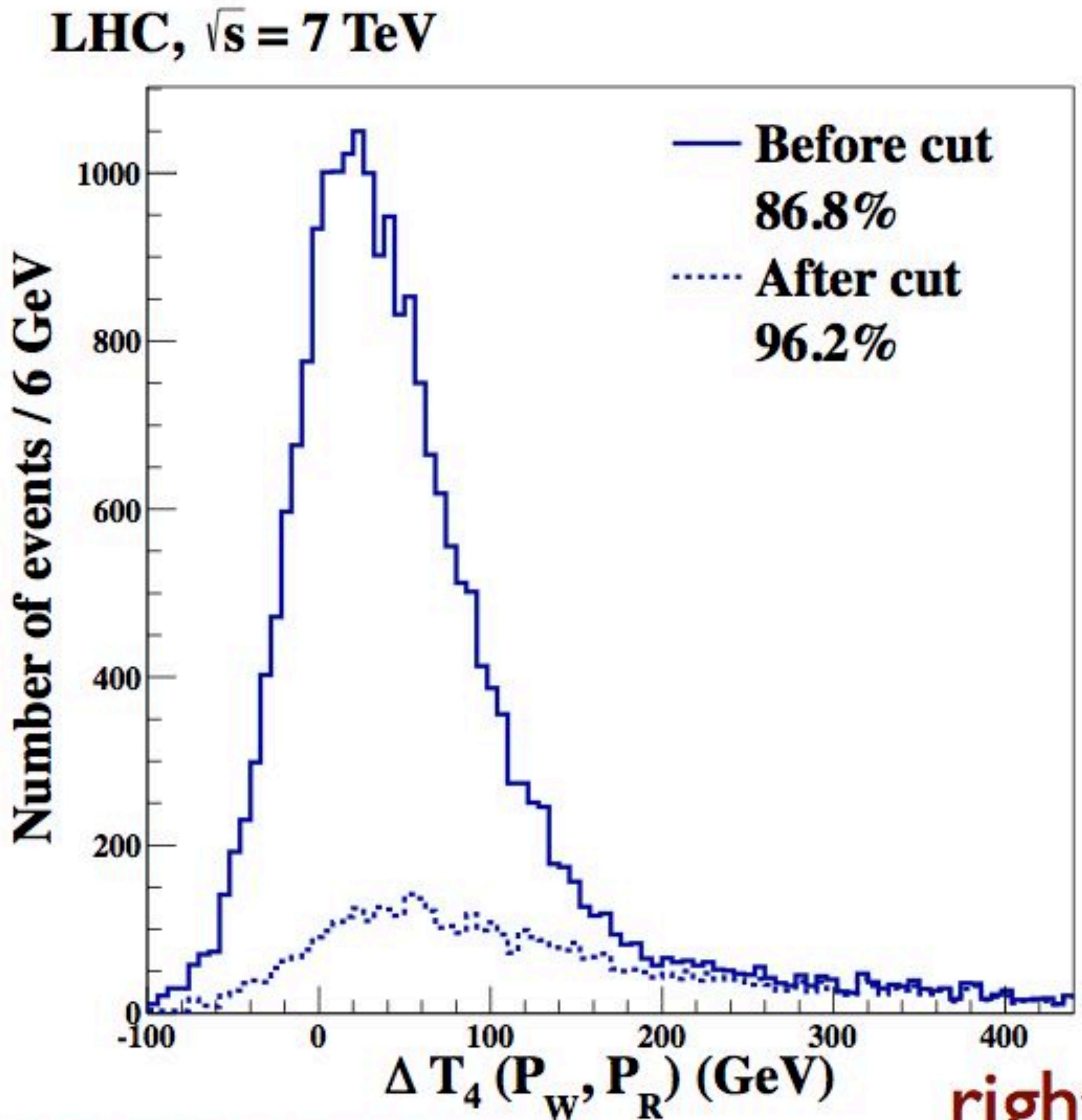
top-pair full system (2b+2l+MET): $M_{T2}^{t\bar{t}}$

Combinatorics in collider signature: test variable



): $M_{T2}^{t\bar{t}}$

Combinatorics in collider signature: test variable



right wrong pairing

$$T_4 \equiv \left| m_W^{\text{maos}} - m_W \right|$$

criterion: $T(P_R) < T(P_W)$.

Combinatorics in collider signature: test variables

$$t_1 + t_2 \to b_1(p^{b_1})l_1(p^{l_1})\nu_1(k_1) + b_2(p^{b_2})l_2(p^{l_2})\nu_2(k_2)$$

$$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$$

$$T_2 \equiv M_{T2}^{t\bar{t}}$$

$$T_3 \equiv |m_t^{\rm maos} - m_t|$$

$$T_4 \equiv |m_W^{\rm maos} - m_W|$$

→ combined method?

$\ast$ take a partition obeying the features as much as possible.

Combinatorics in collider signature: test variables

$$t_1 + t_2 \to b_1(p^{b_1}) l_1(p^{l_1}) \nu_1(k_1) + b_2(p^{b_2}) l_2(p^{l_2}) \nu_2(k_2)$$

$$T_1 \equiv \max\{m_{bl}^{(1)}, m_{bl}^{(2)}\}$$

$$T_2 \equiv M_{T2}^{t\bar{t}}$$

$$T_3 \equiv |m_t^{\rm maos} - m_t|$$

$$T_4 \equiv |m_W^{\rm maos} - m_W|$$

→ combined method?

∗ take a partition obeying the features as much as possible.

➤ select a partition with the smallest value of T_i for the largest number of test variables.

*(~ **89**% efficiency without loss of statistics)*

Combinatorics in collider signature: test variables + event selection cut

wrong partitions: no definite cutoff,
distribution becomes broader for boosted events.

Combinatorics in collider signature: test variables

+ *event selection cut*

wrong partitions: no definite cutoff,
distribution becomes broader for boosted events

Partition-insensitive kinematic variables:

$M_T^{t\bar{t}}$ (transverse mass of top-pair full system)

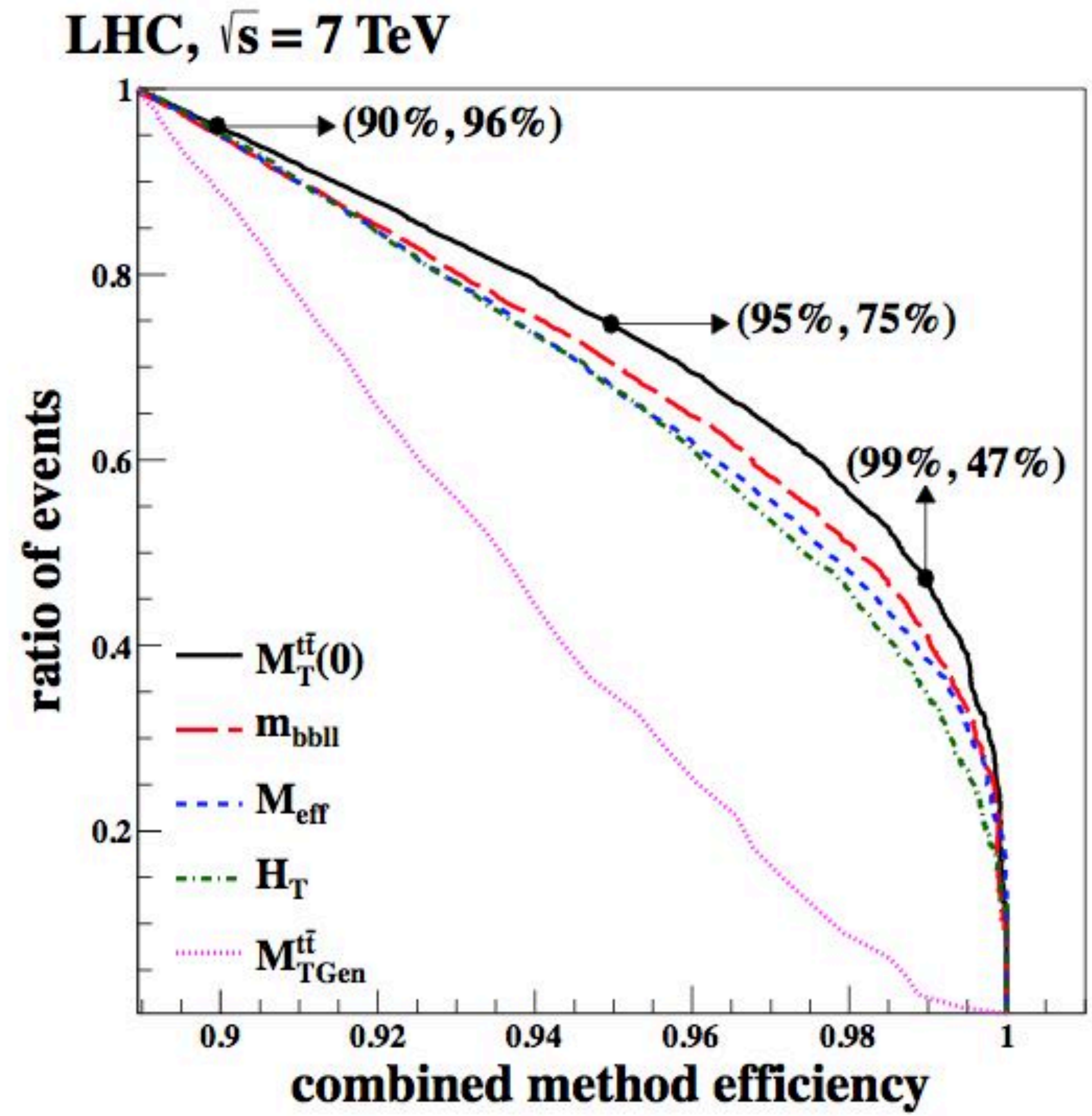
m_V (invariant mass of 2b+2l system)

M_{eff} (scalar sum of all P_T + missing E_T)

H_T (scalar sum of all P_T)

$M_{T\text{Gen}}^{t\bar{t}}$ (smallest M_{T2} of all possible in full system)

Combinatorics in collider signature: test variables + event selection cut



Summary and Outlook

* We proposed a novel method to resolve combinatorial ambiguities in hadron collider events involving two invisible particles in the final state.

* The method is based on the kinematic variables with the assumption of decay topology.

* It can be utilized for new physics processes with pair production and WIMP - in case of pairing and/or ordering ambiguities - (e.g. R-parity conserving SUSY), as well as the dileptonic SM top-pair process.

* For practical use, we will also study the effects of momentum smearing, ISR, and poor mass information.