# End-point contributions

## in exclusive production

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# Outline

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# Approaches to endpoint contributions

- suppression via modified gluon distribution $Q^2 \to z\bar{z}Q^2$. Martin, Ryskin, Teubner (1997)
- suppression by Sudakov formfactor, Botts and Sterman (1989)
- suppression by higher-twist sum, A. Ivanov, R.K. (2003)
- impact factor $\gamma^* \rho_T$ is singularity free, Anikin et al. (2008)

Endpoint enhancement is important in $\sigma_L/\sigma_T$, angular correlations.

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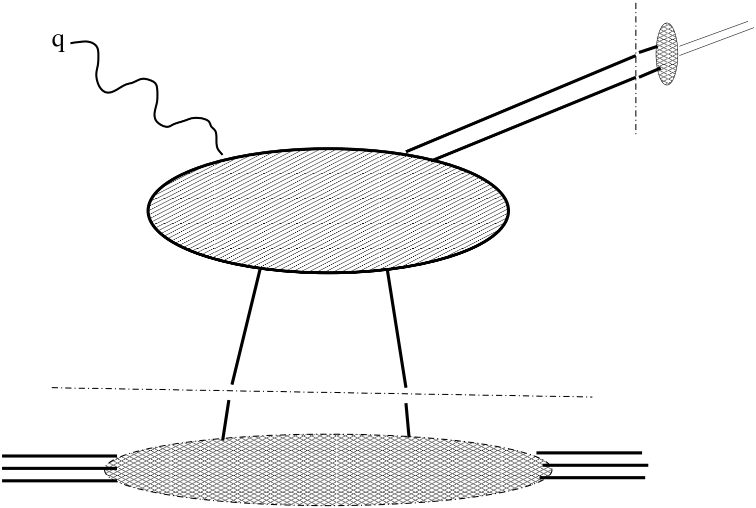
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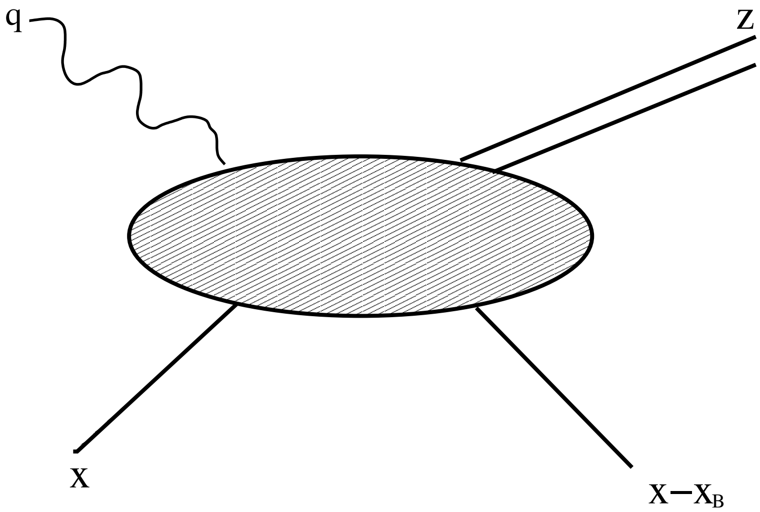

q
Z
X
X−X_B

*collinear kinematics*

$$q = p_2 - x_B p_1 ,$$

$$k_1 = x p_1 , \quad \bar{k}_1 = \bar{x} p_2$$

$$k_2 = z p_2 , \quad \bar{k}_2 = \bar{z} p_2$$

$$z + \bar{z} = 1$$

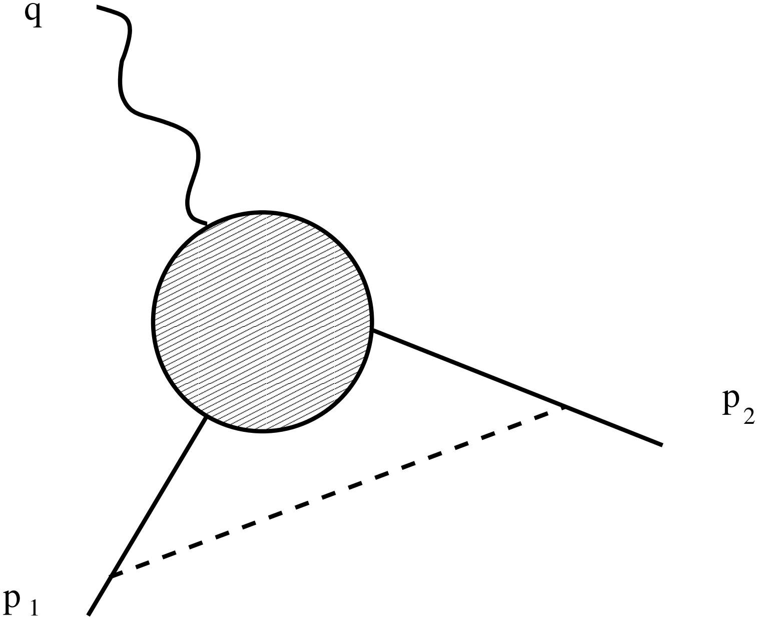
$$x - x_B = \bar{x}$$

$$q^2 = -x_B s , \quad x_B = \frac{Q^2}{s}$$

$$s' = (q + k_1)^2 = (x - x_B) s > 0$$

$$x > x_B$$

$k_1, \bar{k}_1$ and $k_2, \bar{k}_2$ are almost collinear.
Virtual soft-gluon contributions are $\sim \ln^2(k_i k_j)$
Contributions are small at $x \sim \bar{x}$ and $z \sim \bar{z}$
Sizable contributions at $z \to 0$ or $\bar{z} \to 0$.
Sudakov suppression is expected in these regions, if real bremsstrahlung is suppressed.
Real bremsstrahlung at the hard subprocess means DA and GPD contributions with extra soft gluons !
Assumptions about higher twist contributions are involved.




$F(x)$ (double log) contributions with $\ell_\perp < \kappa$.

# Sudakov formfactor

$F(\kappa)$ (double log) loop contributions with $\ell_\perp > \kappa$ universal bremsstrahlung factor:

$$\sim \frac{p_1^\mu}{2p_1 k} \cdot \frac{p_{2\mu}}{2p_2 k} = \frac{1}{\alpha\beta s}$$

loop integral

$$\frac{d^4 k}{k^2} = \frac{s}{2} \frac{d\alpha d\beta d^2\kappa}{(\alpha\beta s + \kappa^2)}$$

$$-g^2 \int F(\kappa) \frac{d\alpha}{\alpha} \frac{d\beta}{\beta} \frac{d^2\kappa}{\alpha\beta s + \kappa^2} \rightarrow$$

$$-g^2 \int F(\kappa^2 = \alpha\beta s) \frac{d\alpha}{\alpha} \frac{d\beta}{\beta}$$

$$= -g^2 \int_{\mu^2}^{Q^2} F(\kappa) \frac{d\kappa^2}{\kappa^2} \int_{\kappa^2}^{Q^2} \frac{d\alpha s}{\alpha s} = -g^2 \int_{\mu^2}^{Q^2} F(\kappa) \frac{d\kappa^2}{\kappa^2} \ln \frac{Q^2}{\kappa^2}$$

first iteration:

$$1 - \frac{1}{2} g^2 \ln^2 \frac{Q^2}{\mu^2}$$

exponentiation:

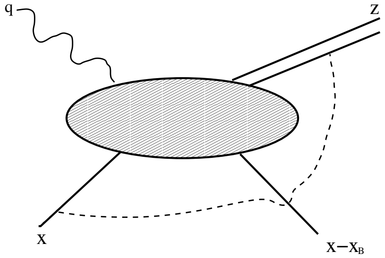
$$F(\kappa) = f_0 - g^2 \int_{\mu^2}^{Q^2} F(\kappa) \ln \frac{Q^2}{\kappa^2} \frac{d^2\kappa}{\kappa^2}$$

$$\frac{d}{d \ln \frac{Q^2}{\mu^2}} F = -g^2 \ln \frac{Q^2}{\mu^2} F$$

$$F = F_0 \exp\left(-\frac{1}{2} g^2 \ln^2 \frac{Q^2}{\mu^2}\right)$$

# Comments

1. bremsstrahlung factorization arises by 1-particle dominance, $(p \pm k)^2 \to 0$.
2. double logs arise due to a log from two Sudakov component integral, i.e. two propagators are near mass shell.
Modification in axial gauges: One pole factor is the axial pole in the gluon propagator.




invariants

$$k_1 \bar{k}_1 = 0, \quad k_2 \bar{k}_2 = 0$$

$$2k_1 k_2 = xzs, \quad 2k_1 \bar{k}_2 = x\bar{z}s$$

$$2\bar{k}_1 k_2 = (x - x_B)zs 2\bar{k}_1 \bar{k}_2 = (x - x_B)\bar{z}s$$

1st iteration, abelian case

$$-\frac{1}{2}\ln^2\frac{k_1 k_2}{\mu^2}-\frac{1}{2}\ln^2\frac{\bar{k}_1\bar{k}_2}{\mu^2}+\frac{1}{2}\ln^2\frac{k_1\bar{k}_2}{\mu^2}+\frac{1}{2}\ln^2\frac{\bar{k}_1 k_2}{\mu^2}=$$

$$-\frac{1}{2}\ln^2\frac{xzs}{\mu^2}-\frac{1}{2}\ln^2\frac{(x-x_b)\bar{z}s}{\mu^2}+\frac{1}{2}\ln^2\frac{x\bar{z}s}{\mu^2}+\frac{1}{2}\ln^2\frac{(x-x_B)zs}{\mu^2}=$$

$$-\ln z\bar{z}\;\ln\frac{x-x_B}{x}\;<\;0$$

formal exponentiation

$$\exp(-g^2\ln z\bar{z}\;\ln\frac{x-x_B}{x})=(z\bar{z})^{g^2\ln(\frac{x}{x-x_B})}$$

*Comments*

- resulting 1-loop contribution negative, suppression of end points.
- result goes beyond double-log approximation, no factorisation, no exponentiation.
- endpoint regularization appears obscure, $\sim \frac{1}{g^2}$
- real bremsstrahlung contributions cannot be neglected, assumptions about higher twist contributions are involved.

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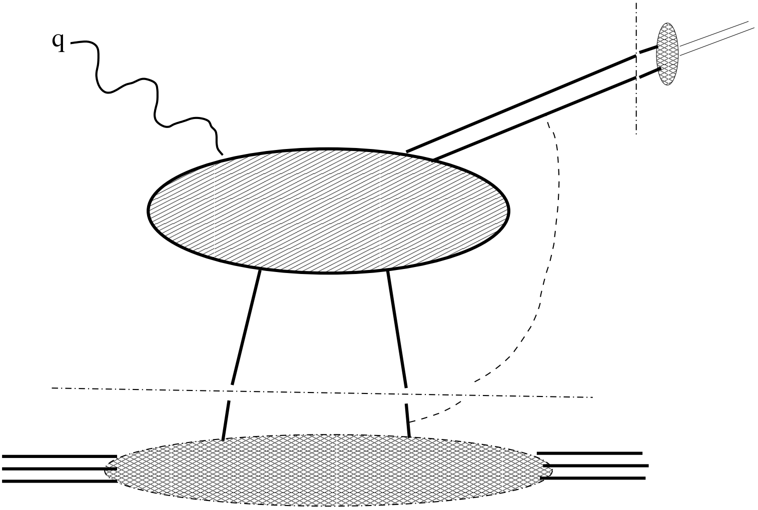
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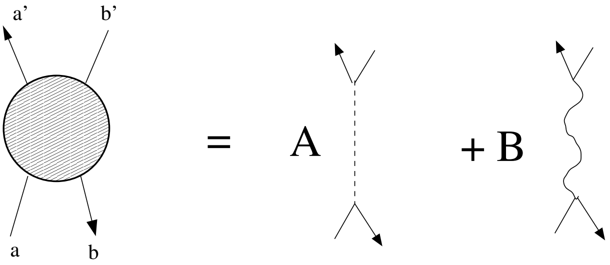
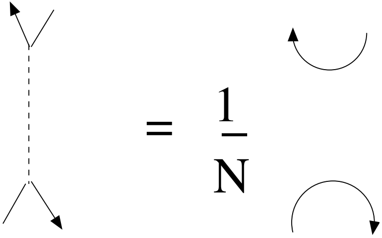
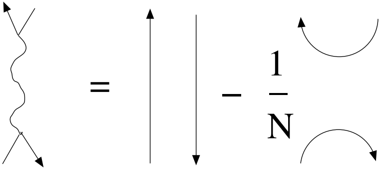
# Colour structure

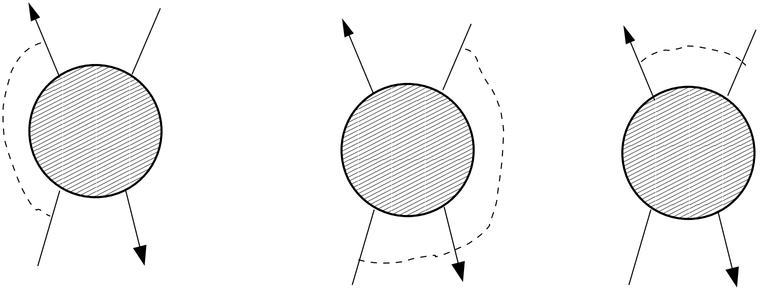





$$= \frac{1}{N}\delta_{ab}\delta_{a'b'}$$




$$= \delta_{aa'}\delta_{bb'} - \frac{1}{N}\delta_{ab}\delta_{a'b'}$$



$\hat{m}_s$ $\hat{m}_u$ $\hat{m}_t$

1-loop contribution:

$$-\hat{m}_s \ln^2 \frac{xzs}{\mu^2} - \hat{m}_s \ln^2 \frac{(x-x_b)\bar{z}s}{\mu^2} + \hat{m}_u \ln^2 \frac{x\bar{z}s}{\mu^2} + \hat{m}_u \ln^2 \frac{(x-x_B)}{\mu^2}$$

$$-\frac{1}{2}(\hat{m}_s + m_u) \ln z\bar{z} \; \ln \frac{x - x_B}{x} +$$

$$\frac{1}{2}(\hat{m}_u - \hat{m}_s)(\ln^2 \frac{xzs}{\mu^2} + \ln^2 \frac{(x-x_b)\bar{z}s}{\mu^2} + \ln^2 \frac{x\bar{z}s}{\mu^2} + \ln^2 \frac{(x-x_B)zs}{\mu^2}$$

$$\hat{m}_s = \begin{pmatrix} 0 & \frac{N^2-1}{2N} \\ \frac{1}{2N} & \frac{N^2-2}{2N} \end{pmatrix} \qquad \hat{m}_s - \hat{m}_u = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N}{2} \end{pmatrix}$$

$$\hat{m}_s - \hat{m}_u + \hat{m}_t = \frac{N^2-1}{2N} I$$

*Discussion*

- full double-log contribution in the octet channel only
- endpoint suppression appears indirectly, both colour channels are involved.
- non-factorizable contributions must be included.

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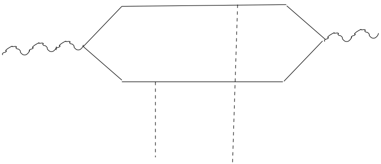
# Endpoint singularities

- arise in collinear kinematics,
- are regularized by higher-twist contributions.

typical situation:

$$\int_0^1 \frac{dz}{z} \frac{1}{Q^2 + \frac{m^2}{z}}$$

example: $\gamma^* p \to \gamma^* p$



*diffractive case*: There is no divergence in impact factor.
*non-diffractive case*: No divergence either, loop contribution is divided into correction to coeff. function and logarithmic contribution to the evolution.

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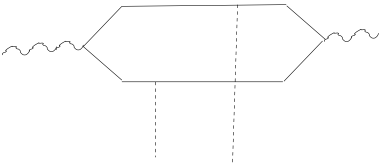

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$$\Phi^{\lambda_i\lambda_f}(\kappa_1,\kappa_2) =$$

$$\int d^2\ell_1 d^2\ell_2 dz \psi_i^{\lambda_i}(l_1,z)\phi^{dipole}(\ell_1,l_2,\kappa_1,\kappa_2)\psi_f^{\lambda_f *}(\ell_2 - zq, z)$$

$$\phi^{dipole}(\ell_1,\ell_2,\kappa_1,\kappa_2) = \alpha_s[\delta^2(\ell_2-\ell_1)+\delta^2(\ell_2-\ell_1+\kappa_1+\kappa_2)-\delta^2(\ell_2-\ell_1+\kappa_1)-\delta^2(\ell_2-\ell_1+\kappa_2)]$$

$\gamma^*$ wave function, perturbative:

$$\Psi^{(\gamma)\lambda}(\ell,z,Q) = e\frac{V^\lambda(\ell,z,Q)}{[Q^2+\frac{|\ell|^2+m_q^2}{z\bar{z}}]}$$

$$V^{(0)} = Q,\ \ V^{(+1)} = \frac{\ell^*}{z},\ \ V^{(-1)} = \frac{\ell}{\bar{z}}$$

vector meson wave function, model

$$\Psi^{V\lambda}(\ell,z) = f_V\frac{V^\lambda(\ell,z,m_V)}{m_V^2}\exp\left[-\frac{|\ell|^2+m_q^2}{z\bar{z}m_V^2}\right]$$

$$\Phi = \Phi_1 + \Phi_{z_0} + \Phi_{\bar{z}_0}.$$

result for $z = O(1)$

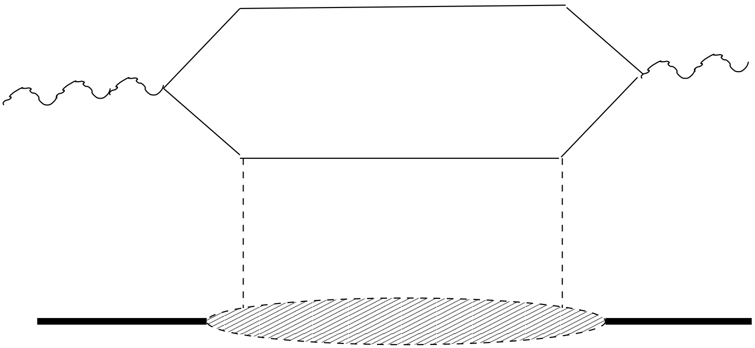
$$\Phi_1^{\lambda_1 \lambda_f}(q,k) = \int_{z_0}^{1-z_0} \varphi_4^{\lambda_i \lambda_f}(q,k) z\bar{z} dz,$$

$$z\bar{z}\varphi_4^{11}(q,k) = C^{11} \frac{k(k+q)^* + k^*(k+q)}{\widetilde{Q}^2}\left(\frac{1}{z\bar{z}} - 2\right).$$

holds for both $\gamma^*\gamma^*$ and $\gamma^* V$.

$$\Phi_{z_0}^{\lambda_1 \lambda_f}(q,k) = C^{11} \frac{k(k+q)^* + k^*(k+q)}{\widetilde{Q}^2} \ln \frac{\tilde{Q}^2 z_0}{|k|^2}$$

endpoint logarithms can be related to parton evolution. Quarks or gluons with $z \to 0$ are not partons of $\gamma^*$, $V$ but rather partons of the target.



The loop contribution can be split in to regular correction to the impact factor /coeff. function and the parton splitting contribution (DGLAP/ERBL).

## parton splitting contribution at the endpoint

$$\Phi^{\gamma,1,1}(k,0)|_{z_0} =$$

$$\int d^2\ell \int_0^{z_0} dz \left\{ \frac{e^2\alpha_S \ell(\ell-\kappa)^* \frac{1}{z^2}}{\left[x_1 s + \frac{|\ell|^2}{z}\right]\left[x_2 s + \frac{|\ell-\kappa|^2}{\bar{z}}\right]} - \ldots(\kappa=0)\ldots \right\}$$

$$= e^2\alpha_S \int \frac{d^2\ell'}{z} \int_0^1 dy \int_0^{z_0} \frac{dz}{z}$$

$$\left\{ \frac{-2\frac{|\ell'|^2}{z}|\kappa|^2 y\bar{y} - \bar{y}|\kappa|^2 s(x_1 y + x_2\bar{y}) + \mathcal{O}(|\kappa|^4)}{\left[s(x_1 y + x_2\bar{y}) + \frac{|\ell'|^2}{z}\right]^3} \right\}$$

restore the integration over the $p_1$ momentum fractions

$$\int_0^1 s d\beta_1 \int_0^{\beta_1} d\beta_\ell \ldots z\delta(z(\beta_1-\beta_\ell)s-|\ell'|^2) \;\; \delta((\beta_\ell-x_1)s-|\ell'+\kappa|^2$$

$$z\delta((z-\alpha_1)(\beta_1-\beta_\ell)s-|\ell-\kappa|^2-m_q^2)$$

$$\bar{z}\delta(\bar{z}(\beta_\ell-x_1)s-|\ell|^2-m_q^2)$$

The helicity amplitude contribution:

$$M^{\gamma,11} = e^2 \int_{m_V^2}^{z_0 Q^2} \alpha_S(|\ell|^2) \frac{d|\ell|^2}{|\ell|^2} \int_0^1 d\beta_1 d\beta_2 \delta(x_1 - x_2 - \beta_1 + \beta_2)$$

$$P(x_1, x_2; \beta_1, \beta_2) G_g(\beta_1, \beta_2; q = 0, |\ell|^2)$$

$$P(x_1, x_2; \beta_1, \beta_2) = \frac{\Theta(\beta_1 - x_1)}{\beta_1 \beta_2}(1 + \mathcal{O}(\beta_1 - x_1))$$

# Conclusions

- singularities at endpoints are due to collinear approximation
- endpoint enhancement is phenomenologically significant
- critical view on the regularization by Sudakov formfactor
- suppression by bremsstrahlung involves assumption on *higher twist*
- meson wave function involves regularization by a series of higher twist contributions.
- quark or gluon with momentum fraction $z \to 0$ is not a parton of the meson, rather a parton of the target
- meson wave function: relation to non-perturbative correlators like DA not explicit, but qualitative features reproduced

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