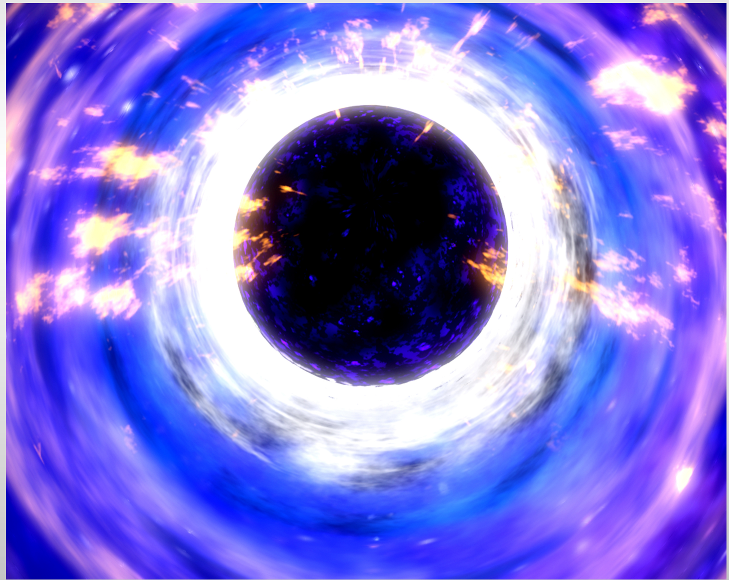
Black Holes and Inflation



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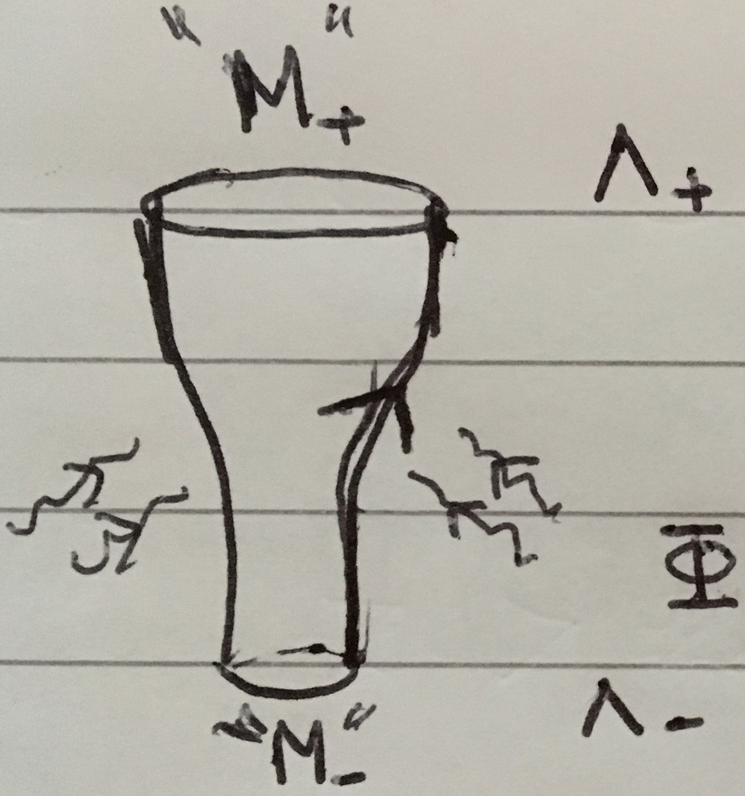
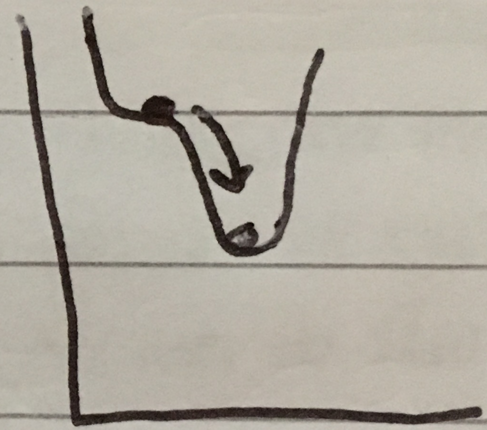
Motivation came from Thermodynamics with variable Lambda





Inflation provides a set-up in which Lambda varies slowly

Thermodynamics

A black hole has temperature, entropy, and satisfies a first law:

$$\delta M = T \delta S$$

Can derive this by varying the Schwarzschild potential:

$$\delta f(r_+ + \delta r_+) = -\frac{2\delta M}{r_+} + \frac{2M}{r_+^2}\delta r_+ = 0$$

But we are used to

$$dU = TdS - pdV$$

Thermodynamics and Lambda

Now vary the Schwarzschild potential with lambda:

$$\delta f(r_+ + \delta r_+) = -\frac{\delta\Lambda}{3} r_+^2 - \frac{2\delta M}{r_+} + f'(r_+)\delta r_+ = 0$$

Rearrange to

$$\delta M = T\delta S - \frac{4\pi r_+^3}{3}\delta\left(\frac{\Lambda}{8\pi}\right)$$

Enthalpy

Thermodynamic Volume

Thermodynamic Pressure

Varying Lambda

The idea of a varying Lambda is very familiar – in slow roll inflation, lambda varies gradually, while our universe is quasi-de Sitter.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left[\cancel{\frac{\dot{\phi}^2}{2}} + W(\phi)\right]$$

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -\frac{\partial W}{\partial \phi}$$

Small slow-roll parameters ensure that inflation in maintained:

$$\varepsilon = \frac{M_p^2}{2}\frac{W'^2}{W^2} \qquad \Gamma = 2M_p^2 \frac{W''}{W}$$

Add Black Hole

But a key difference is that our geometry is not explicitly time dependent – so what does “slow roll” mean?

$$V_{SDS} = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

Without the black hole, the transformation to cosmological time is nontrivial:

$$\tau_{cos} = t_{sds} + \frac{1}{H_0} \log(1 - H_0^2 r_{sds}^2)$$

And, apart from SDS, difficult to find time dependent black hole solutions

With Black Hole

With a black hole, intuition is that the geometry is approximately SDS, the scalar still slow-rolls, but that this produces a sub-leading effect on the background black hole geometry. The spacetime slides from one Lambda to a lower one, and the black hole accretes a little mass.

$$\phi = \phi_0 + \delta\phi_{SR}$$

$$g_{\mu\nu} = \underset{\text{SDS}}{g_{0\,\mu\nu}} + \underset{\mathcal{O}\left(\delta\phi_{SR}\right)^2}{\delta g_{SR\,\mu\nu}}$$

Equations

To follow event horizons, use null coordinates:

$$ds^2 = 4e^{2\nu}\sqrt{\frac{B_0}{B}}dUdV - Bd\Omega_{\mathrm{II}}^2$$

See how leading order is Φ rolling on black hole background

$$\phi_{,UV} = -W_{,\phi}(\phi)\sqrt{\frac{B_0}{B}}e^{2\nu} - \frac{1}{2B}\left(B_{,U}\phi_{,V} + B_{,V}\phi_{,U}\right)$$

$$B_{,UV} = 2\left(\frac{W(\phi)}{M_p^2}B^{1/2} - B^{-1/2}\right)e^{2\nu}B_0^{1/2}$$

$$\nu_{,UV} = \frac{1}{2}\left(\frac{W(\phi)}{M_p^2}B^{-1/2} + B^{-3/2}\right)e^{2\nu}B_0^{1/2} - \frac{\phi_{,U}\phi_{,V}}{2M_p^2}$$

$$B_{,VV} = 2\nu_{,V}B_{,V} - B\phi_{,V}^2/M_p^2$$

$$B_{,UU} = 2\nu_{,U}B_{,U} - B\phi_{,U}^2/M_p^2$$

Background

Starting point is constant phi, and equations integrate up to give SDS in different gauge

$$B = B\left[F(V) + G(U)\right] \qquad e^{2\nu} = F'G'B'$$

$$B' = \frac{4W_0}{3M_p^2}B^{3/2} - 4B^{1/2} + \mu$$

$$\propto GM$$

Physically, easiest to set $B=r^2$. U,V can be Kruskals at each horizon.

$$F, G \leftrightarrow \mp\frac{(t \pm r^\star)}{4}$$

Scalar Field Eqn

In terms of the (shifted) null coordinates, e.o.m for phi is

$$\frac{\phi_{,FG}}{V_{SDS}[r]} - \frac{2}{r}\left(\phi_{,F} + \phi_{,G}\right) = 4\frac{\partial W}{\partial \phi}$$

Motivated by cosmological solution, look for a parameter, x, s.t. phi depends on x, at least to leading order slow-roll.

$$x = t + \xi(r)$$

Find $$x = t - r^{\star} + \frac{1}{\kappa_h}\ln\left|\frac{r - r_h}{r_h}\right| + \ldots$$

At r_h a fn of v, and at r_c a fn of U

Phi Equation

The phi equation is a standard slow-roll type

$$3\gamma\phi'(x) = -\frac{\partial W}{\partial \phi}$$

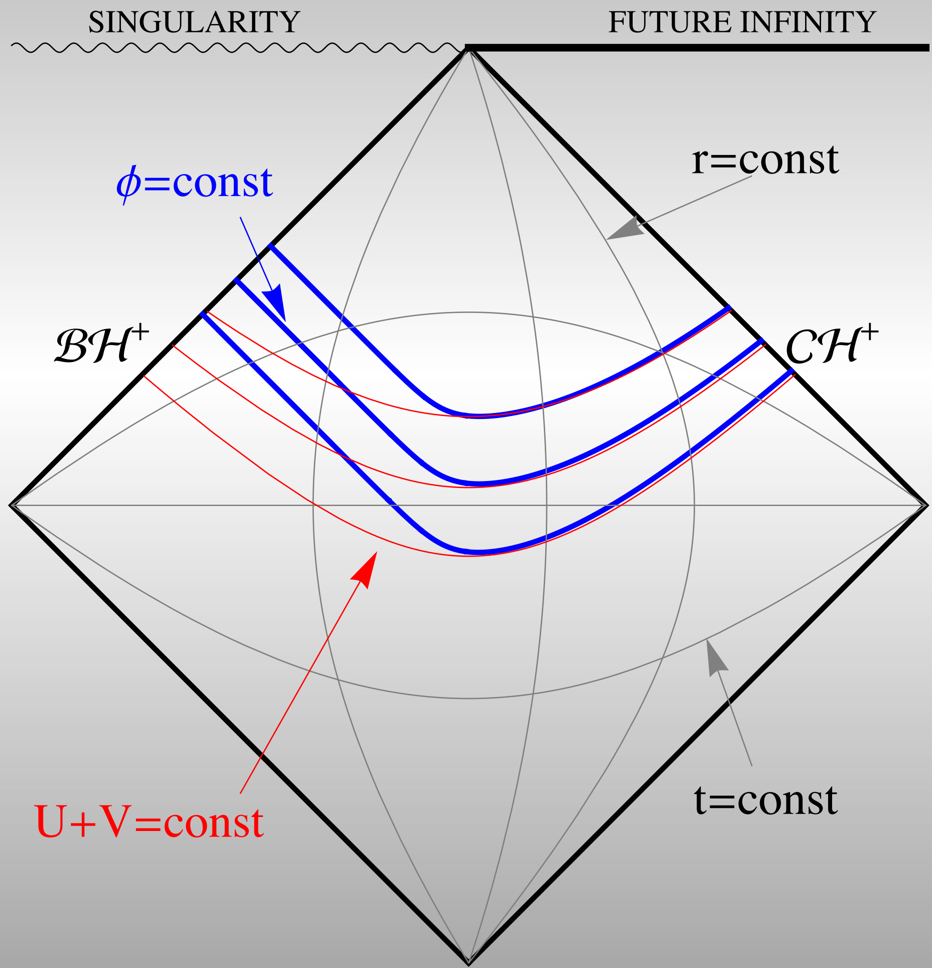
but with friction parameter modified from H:

$$\gamma = \frac{r_c^2 + r_h^2}{r_c^3 - r_h^3} = \frac{A_{TOT}}{3V}$$

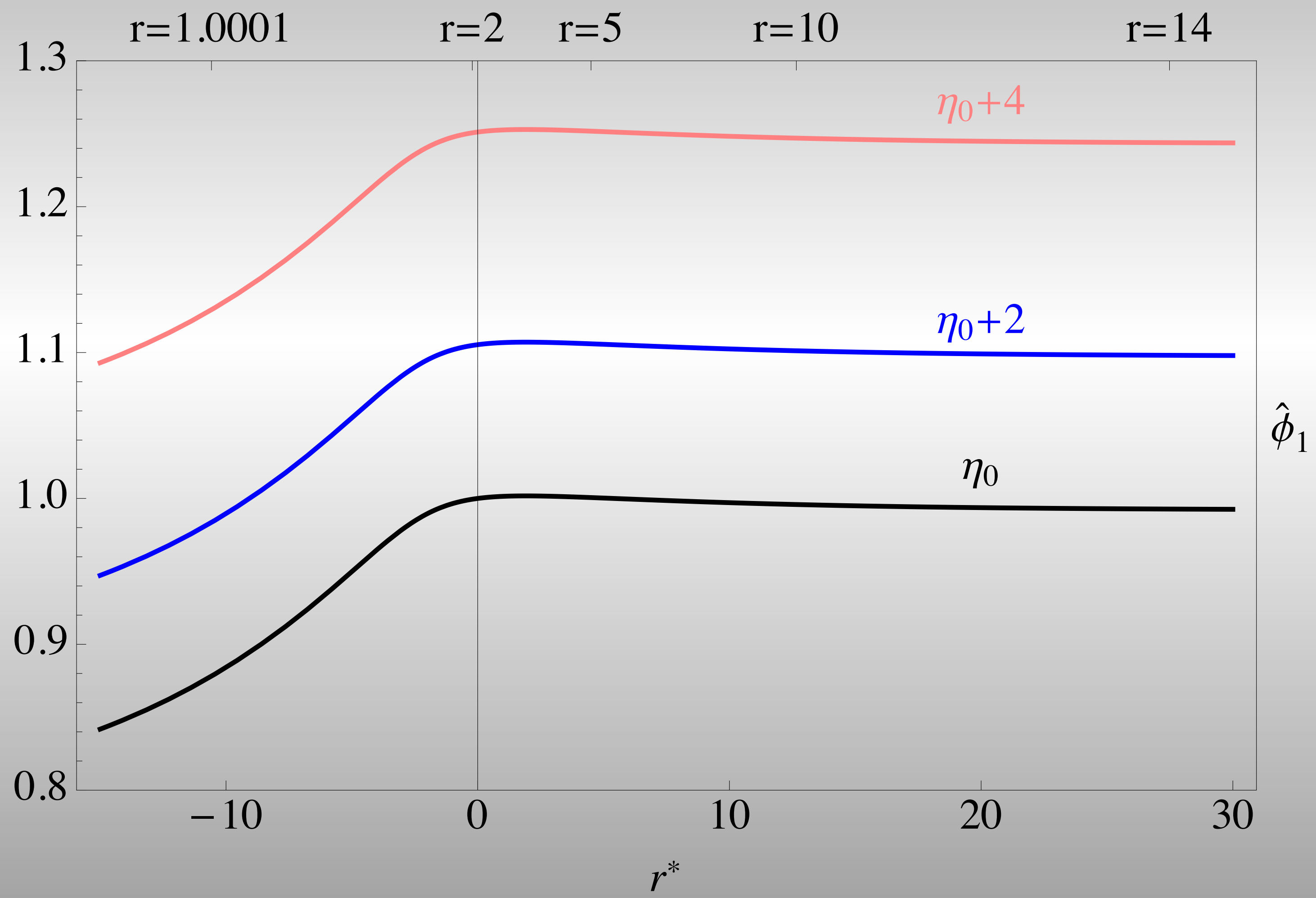
Physical effect of black hole is to add friction to roll, or to slow down the scalar.

Phi Profile

And spatial profile is nontrivial…



Phi behaviour



Horizon Growth

Although the full back-reaction is messy, the solution at each horizon is simpler, e.g. black hole horizon:

$$\delta B = -\frac{B_{0h}}{3\gamma\kappa_h M_p^2} v \int_v^\infty \left(W[\phi(v')] - W[\phi(0)]\right) \frac{dv'}{v'^2}$$

Shows teleological behaviour of horizon, total area shift of

$$\delta A_h = \frac{A_h}{3\gamma\kappa_h M_p^2}(W_i - W_f)$$

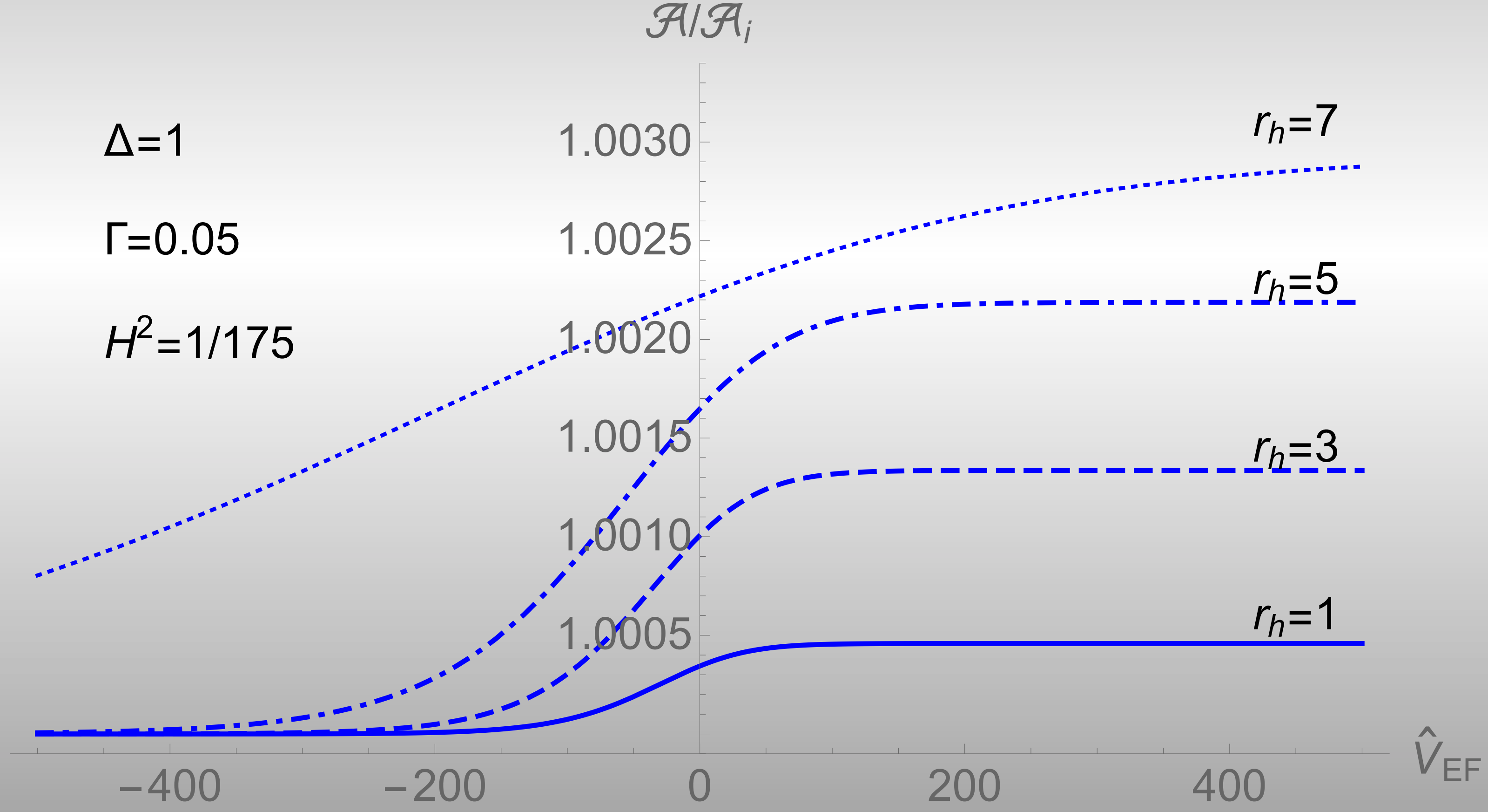
Note

$$\frac{\delta A_h/A_h}{\delta A_c/A_c} = \frac{|\kappa_c|}{\kappa_h} < 1$$

(fractional change in cosmological horizon area greater than that of black hole).

Horizon Growth

From test case covered later, can compute the exact leading order horizon backreaction:

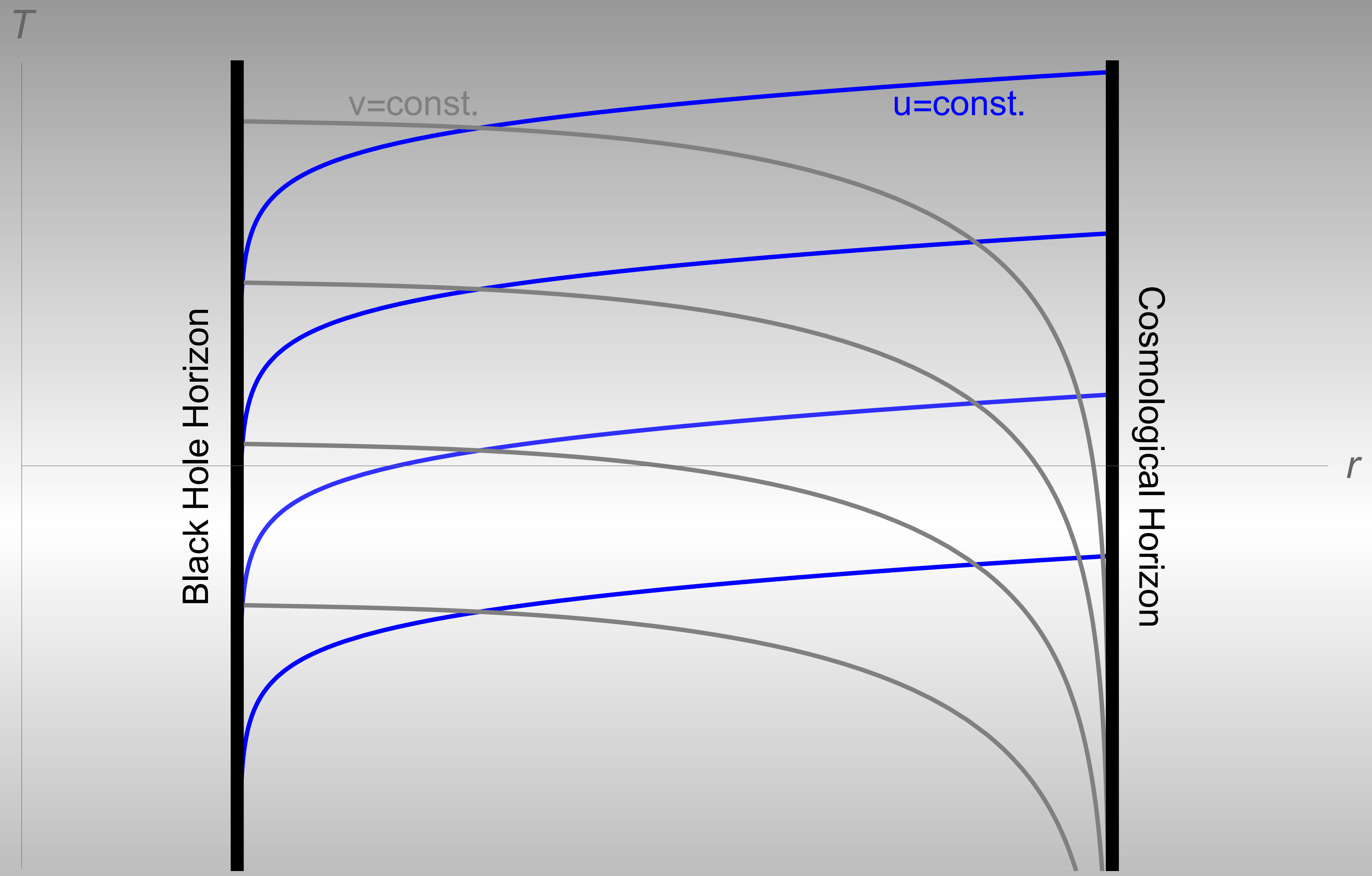


Back To Scalar

Can find thermodynamics of the black hole (more later!), but go back to the dependence of the scalar, and relabel x-parameter as T.

$$T = t + \xi(r)$$

T is constructed so that phi is regular at both horizons, with only in(out) going modes at black hole (cosmological) horizon.



T looks like an Eddington-Finkelstein coord on each horizon, at r_h a fn of v, and at r_c a fn of U.

Back-Reaction

Given this Eddington-Finkelstein behaviour, look at SDS metric in (T,r) coords:

$$ds^2 = f_{sds}\, dT^2 - 2 f_{sds}\xi'\, dT dr - \frac{1}{f_{sds}}\left(1 - \left(f_{sds}\xi'\right)^2\right) dr^2 - r^2 d\Omega^2$$

Because of the behaviour of xi, this is regular at both (future) horizons, with T being more obviously related to cosmological time at large r.

With this input, keep T as a coordinate, and make the metric Ansatz:

$$ds^2 = f(r,T)\,dT^2 - 2h(r,T)\,dT dr - \frac{dr^2}{f}\left(1 - h^2\right) - r^2 d\Omega^2$$

The energy momentum of the scalar has 2 independent cpts:

$$T_{TT} = \left(W(\phi) + \frac{1+h^2}{2f}\dot{\phi}^2\right)|g_{TT}|\,,$$

$$T_{ab} = \left(-W(\phi) + \frac{1-h^2}{2f}\dot{\phi}^2\right)g_{ab}$$

Which we relate to the Einstein tensor:

$$G_{TT} = \left[\frac{1}{r^2}(1 - f - rf') - \frac{h\dot{f}}{rf}\right] |g_{TT}|$$

$$G_{rr} = \left[-\frac{1}{r^2}(1 - f - rf') + \frac{h\dot{f}}{rf} + \frac{2\dot{h}}{r(1-h^2)}\right] g_{rr}$$

$$G_{rT} = \left[-\frac{1}{r^2}(1 - f - rf') - \frac{(1-h^2)\dot{f}}{rhf}\right] g_{rT}$$

$$G_{\theta\theta} = \left[\frac{f''}{2} + \frac{f'}{r} - \frac{h'\dot{f}}{2f} + \frac{\dot{h}f'}{2f} + \frac{\dot{h}}{r} + \dot{h}' + \frac{1}{2}\left(\frac{(h^2-1)}{f}\right)^{\cdot\cdot}\right] g_{\theta\theta} = \frac{G_{\phi\phi}}{\sin^2\theta}$$

? Slow Roll ?

Need to have control of the slow-roll approximation to identify the key dependences in these equations.

The scalar equation is straightforward to see,

$$\frac{1-h^2}{f}\ddot{\phi} - \frac{(r^2 h)'}{r^2}\dot{\phi} = -W'(\phi)$$

But the equivalent of Friedmann is:

$$\left[\frac{1}{r^2}(1-f-rf') + \frac{(1-h^2)\dot{f}}{rhf}\right] = \frac{1}{M_p^2}\left(W(\phi) - \frac{1-h^2}{2f}\dot{\phi}^2\right)$$

Slow Roll with a Black Hole

Taking the same general slow roll requirements, these now depend on position:

$$\frac{1-h^2}{f}\dot{\phi}^2 \ll W \quad , \qquad \frac{1-h^2}{f}\ddot{\phi} \ll \frac{1}{r^2}\left|\left(r^2 h\right)' \dot{\phi}\right|$$

As usual in slow-roll, take background values of metric functions, and can bound these r-dependent background functions to the usual slow roll type parameters

$$\varepsilon = M_p^2 \frac{W'^2}{W^2} \ll 1 \qquad \Gamma = M_p^2 \frac{W''}{W} \ll 1$$

This allows us to solve the Einstein equations to leading order in the slow-roll parameters.

- TT + Tr: $$\dot{f} = -rh\,\frac{\dot{\phi}^2}{M_p^2}$$

implies

$$f(r,T) = f_0(r) + \delta f(r,T) - rh_0 \int \frac{\dot{\phi}^2}{M_p^2}$$

Where δf is order εΓ, but slowly varying. We can then integrate the φ kinetic energy:

$$\int_{T_0}^{T} \dot{\phi}^2 dT' = -\frac{1}{3\gamma}\,\{W(\phi(T)) - W(\phi(T_0)\} = -\frac{\delta W}{3\gamma}$$

And end up with a familiar expression:

$$\boxed{f(r,T) = 1 - \frac{\Lambda(T)}{3M_p^2}r^2 - \frac{2GM(T)}{r} + \delta f}$$

With

$$\Lambda(T) = W[\phi(T)] \qquad\qquad M(T) = M_0 - 4\pi\beta\frac{\delta W}{3\gamma}$$

A somewhat finicky argument shows that delta f is transient, and of sub-leading order ($\varepsilon\Gamma$) to the changes in Λ and M.

We can solve for h(r,T) as well, and we find that to leading order, for a slow roll scalar the black hole geometry takes its "Schwarzschild" form in the scalar T-coordinate (regular on both horizons) but with Λ and M now time varying. We get a remarkably simple expression for the time-dependence of the horizon areas:

$$\dot{A}_h = \frac{A_h}{|\kappa_h|} \frac{\dot{\phi}^2}{M_p^2}$$

Thermodynamics

We can find exact, differential forms of the various thermodynamic first laws:

- De Sitter patch:

$$|\kappa_b|\dot{A}_b + |\kappa_c|\dot{A}_c + V\dot{\Lambda} = 0$$

- Black hole first law:

$$\frac{\dot{M}}{M_p^2} - |\kappa_b|\dot{A}_b + V_b\dot{\Lambda} = 0$$

Which of course begs the question of temperature..

Dynamical Temperature

Hayward et al suggested a dynamical temperature

$$\kappa_{dyn} = \frac{1}{2} \star d \star dr$$

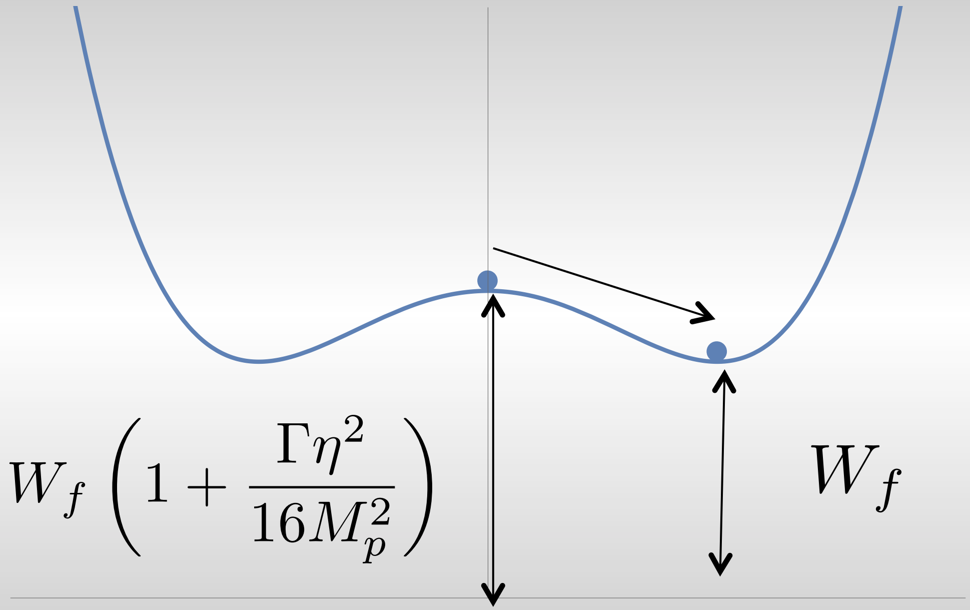
Which we can calculate for our solution

$$\kappa_{dyn}(T) = (f' + \dot{h}) = \kappa_b(T) + \mathcal{O}(\varepsilon\Gamma)$$

i.e. the instantaneous temperature of the time-dependent SdS potential.

Explicit Example

Double well potential



Cosmological soln:

$$\phi^2 = \frac{\eta^2}{2}\left[1+\tanh\left(\frac{H_f\Gamma}{4}\tau\right)\right]$$

At black hole horizon in terms of E-F advanced time:

$$\kappa_h T \simeq \ln(2\kappa_h v) = \ln \hat{v}$$

And write $a = \dfrac{\Gamma H_i^2}{2\gamma\kappa_h}$ so that $\phi^2 = \eta^2 \dfrac{\hat{v}^a}{1+\hat{v}^a}$

Then can extract the behaviour of the horizon, directly depending on the gravitational strength of the scalar

$$\Delta = \frac{\eta^2}{M_p^2}$$

Horizon Growth

Horizon growth depends primarily on Δ but the rate of growth determined by the slow roll friction parameter

$$\mathcal{A} = 4\pi B = \mathcal{A}_0 \left(1 - \frac{a\Delta\hat{v}}{8} \mathcal{I}[\hat{v}, a] \right)$$

(I is the integral in the variation of B)

$$\begin{aligned} \mathcal{I}[\hat{v}, a] &= -\int_{\hat{v}}^{\infty} \frac{y^a(2+y^a)dy}{y^2(1+y^a)^2} \\ &= -\frac{(1+a(1+\hat{v}^a))}{a\hat{v}(1+\hat{v}^a)} + \frac{1+a}{a\hat{v}} \left(1 - {}_2F_1\left[1, \frac{1}{a}; \frac{1+a}{a}; -\hat{v}^{-a}\right] \right) \end{aligned}$$

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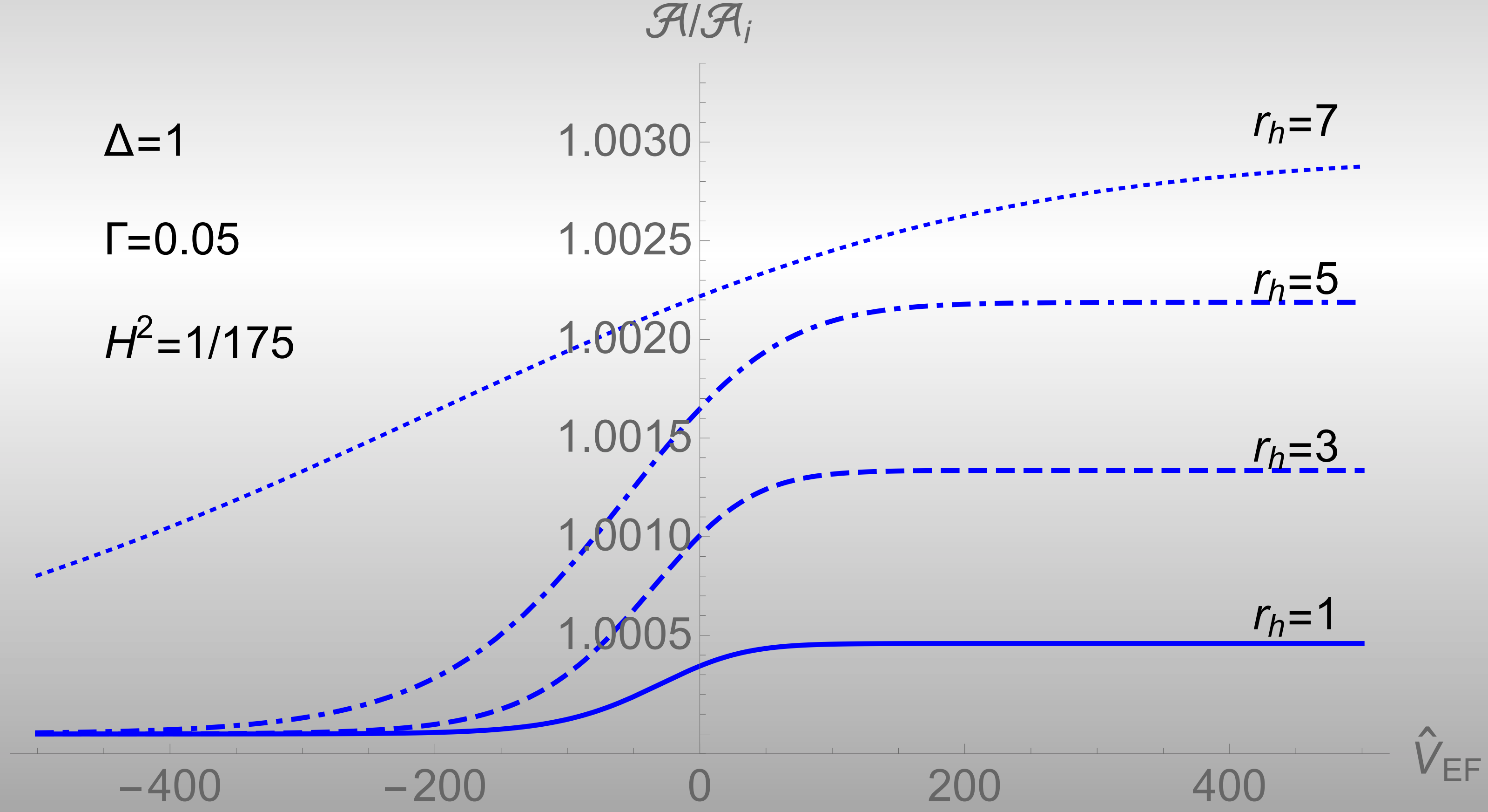
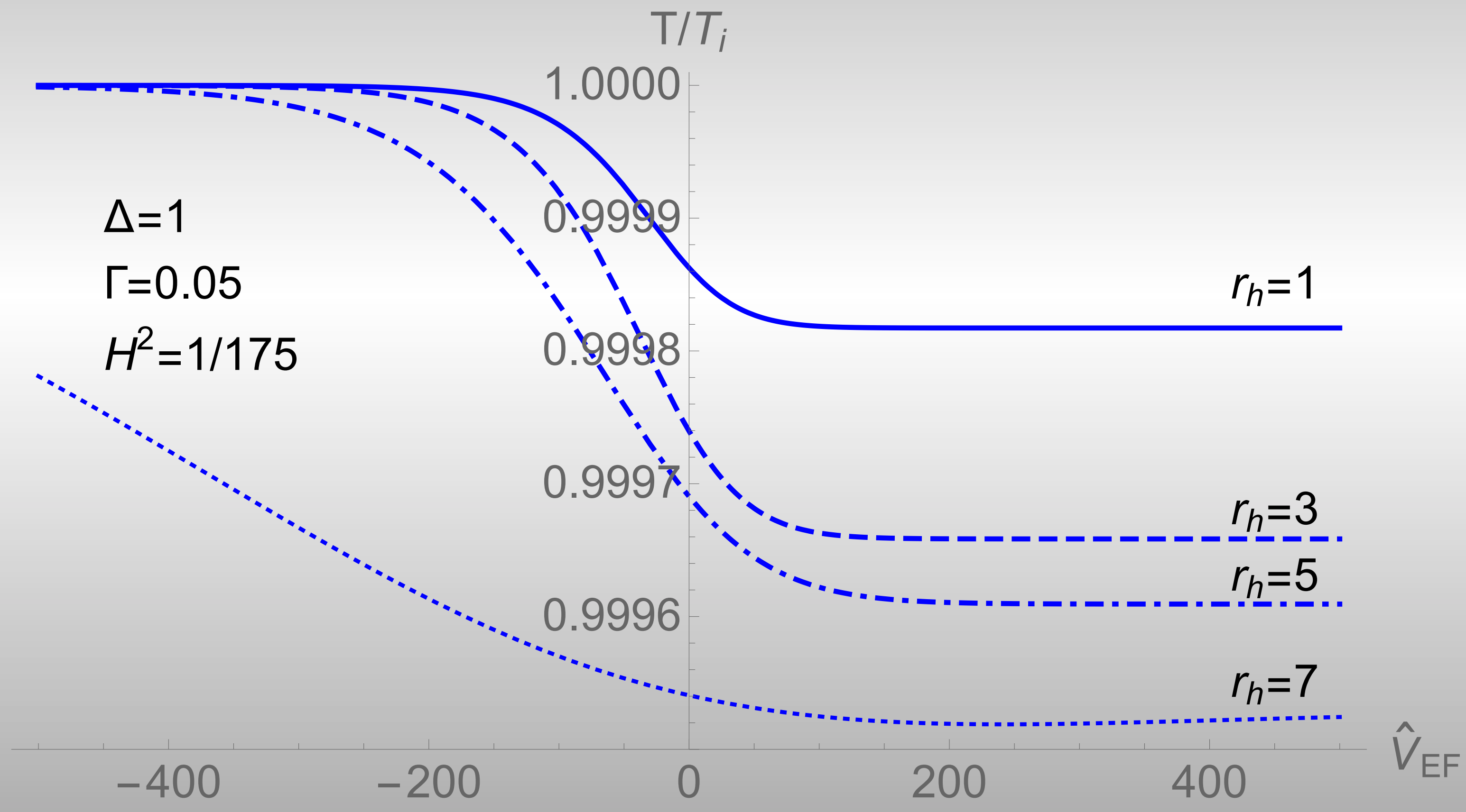


Dynamical T:

And temperature variation:



Summary

- Have generalised slow-roll description to non-homogeneous black hole background.
- The friction parameter for the scalar is increased by the black hole
- Checked the first law – holds dynamically during the flow.
- Explored dynamical temperature
- The black hole geometry is to a very good approximation quasi-Schwarzschild de Sitter.