

Higgs - Induced Lepton Flavor Violation

LPT - Orsay, 8/12/2011

(based on arXiv:1111.1715
with O.Lebedev, J.-h. Park)

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Outline

- Introduction - fermion masses in the SM
- Higgs - dependent Yukawa couplings
- LFV constraints @ 1-loop and 2-loops
- Focus on dimension - 6 operators
- Reproducing the lepton mass hierarchy
- Conclusions

Introduction - the fermion masses in the SM

- In the Standard Model of particle physics, fermion masses are due to the interactions among fermion fields and the Higgs scalar doublet through the Yukawa prescription:

$$\mathcal{L}_Y = Y_{ij}^u Q_{iL} u_j^c H - Y_{ij}^d Q_{iL} d_j^c H^c - Y_{ij}^l L_{iL} e_j^c H^c + h.c.$$

Some consequences :

- 1) Mass and neutral current interaction terms can be diagonalized simultaneously. There are no FCNC's.
- 2) In the quark sector there are charged FCC's (CKM matrix).
- 3) There are no sources of Lepton Flavor Violation (LFV).

- One important remark:

In the SM it is possible to *measure* the Yukawa couplings. There is no way to *predict* their values : they are free parameters. In principle, since they are dimensionless quantities, one would expect them to be O(1).

...but it turns out that the values of the Yukawa couplings span **6 orders of magnitude**, kind of weird for terms seemingly of the same nature! In fact, the only O(1) Yukawa is the t-quark one...

Introduction - efforts towards solving the problem

Much effort has been devoted in the literature in order to explain the pattern of the fermion masses and mixings. Most typical models (not necessarily addressing exactly the same set of questions) are based on :

- **flavor symmetries** (discrete or continuous) : impose some pattern for the Yukawa matrices.
- **extra dimensions** : explain (or interpret) fermion mass hierarchy geometrically (e.g. localize fermions at different positions in the bulk).
- **GUTs** : Depending on the specific construction, they can predict several parameters (occasionally some relations can even kill GUTs and might be unwanted!).
- **Froggatt - Nielsen mechanism** : Assume some continuous $U(1)$ symmetry, distinguishing the fermions. Break the symmetry through some "flavon" scalar S . Communicate the breaking to the fermions through different powers of some parameter $\varepsilon = \langle S \rangle / M_*$ (often $\sim O(0.22)$, the Cabibbo angle), with M_* being associated with some new (usually) high physics scale. The hierarchies are then explained through the dependence on different powers of ε , which acts as a suppression factor.

HDYC : another option

→ What if the Yukawa couplings depend on the Higgs field itself?

(Babu, Nandi - hep-ph/9907213, Giudice, Lebedev - arXiv:0804.1753)

- Assume an effective theory valid below some new physics scale M in which the Yukawas are functions of the Higgs field. Then, they can generically be expanded as

$$Y_{ij}(H) = \sum_{n=0}^{\infty} c_{ij}^{(n)} \left(\frac{H^\dagger H}{M^2} \right)^n$$

- If we assume that the coefficients c_{ij}^n vanish up to a generation-dependent order n , the effective Yukawa Lagrangian can be written as

$$\mathcal{L}_Y = Y_{ij}^u(H) Q_{Li} u_{Rj} H - Y_{ij}^d(H) Q_{Li} d_{Rj} H^c - Y_{ij}^l(H) L_{Li} e_{Rj} H^c + \text{h.c.}$$

Where

$$Y_{ij}^{u,d,l}(H) = c_{ij}^{u,d,l} \left(\frac{H^\dagger H}{M^2} \right)^{n_{ij}^{u,d,l}}$$

→ Then, upon EWSB the Yukawas receive contributions scaling as powers of

$$\epsilon = v^2/M^2$$

HDYC : some non-LFV consequences

Assuming HDYC's has some interesting phenomenological consequences :

- Drastic modification of the Higgs boson couplings. For specific choices we can have :

$$\frac{\Gamma(h \rightarrow b\bar{b})}{\Gamma(h \rightarrow b\bar{b})_{SM}} = \frac{\Gamma(h \rightarrow c\bar{c})}{\Gamma(h \rightarrow c\bar{c})_{SM}} = \frac{\Gamma(h \rightarrow \tau^+\tau^-)}{\Gamma(h \rightarrow \tau^+\tau^-)_{SM}} = 9, \quad \frac{\Gamma(h \rightarrow \mu^+\mu^-)}{\Gamma(h \rightarrow \mu^+\mu^-)_{SM}} = 25$$

- At low masses there can be a significant reduction of $h \rightarrow \gamma\gamma$

- $h \rightarrow WW$ becomes dominant for larger masses than in the SM.

- Appearance of flavor - violating Higgs decays.

(e.g. Giudice, Lebedev - arXiv:0804.1753)

- Tree-level FCNC effects are strongly model-dependent and potentially very dangerous! However, they can be suppressed so as to fulfill current experimental bounds (for a specific construction see, e.g. Giudice, Lebedev - arXiv:0804.1753).

- New sources of CP - violation appear, which can find application in EW baryogenesis (Lebedev - arXiv:1011.2630).

- The quark sector sets the new physics scale M at 1-2 TeV (reminds us of something?).

HDYC : the lepton sector

The suppression of the higher - dimensional operators by a new physics scale M could make us expect that its impact might be relatively small...

However, we should note that the smallness of the lepton sector Yukawa couplings can make these contributions non-negligible!

In particular, we shall see that the mass and interaction matrices are misaligned in flavor space, which can lead to Lepton Flavor Violation mediated by the SM Higgs boson.

We shall take three steps :

- 1) Compute model - independent bounds on the flavor violating couplings of the Higgs boson.
- 2) Examine the impact of dimension - 4 and 6 operators only : constrain NP scale, rotation angles.
- 3) Examine Yukawa textures (patterns) which can reproduce the lepton mass hierarchy.

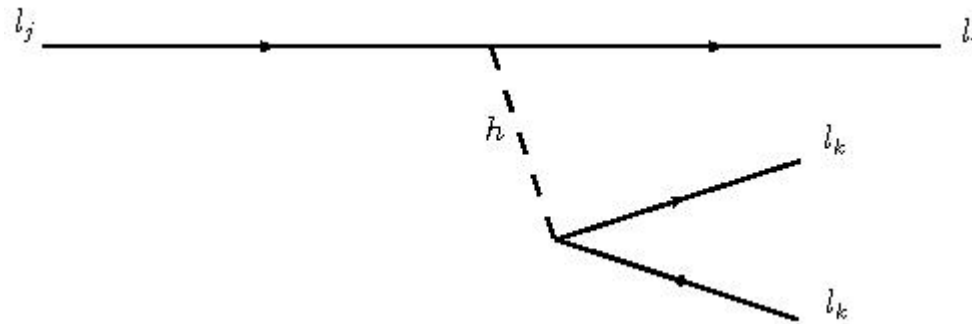
Bounds on Higgs couplings

The hll interaction Lagrangian is :

$$\Delta\mathcal{L} = -\frac{y_{ij}}{\sqrt{2}} h \bar{l}_i P_R l_j + \text{h.c.}$$

which induces contributions to

- flavor - changing three-body decays (e.g. $\mu \rightarrow eee$)



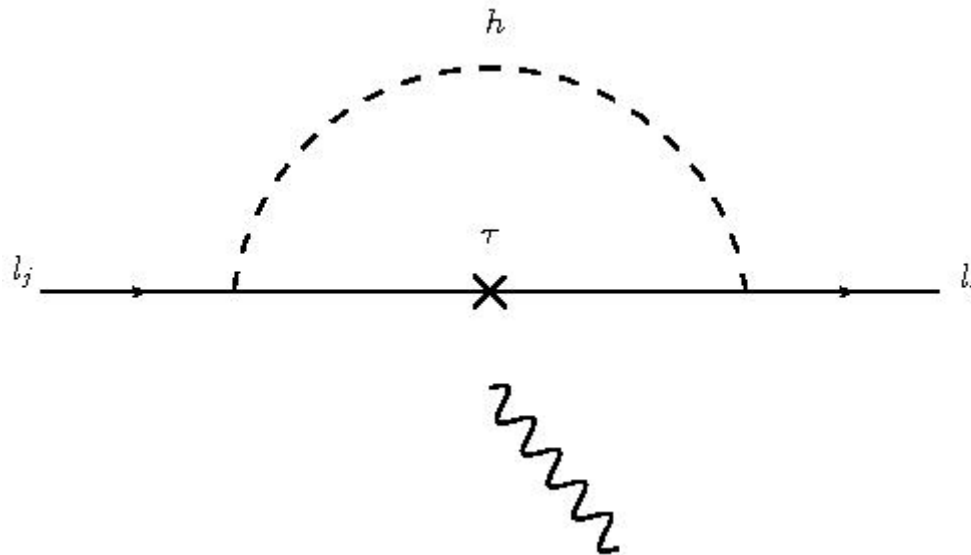
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- radiative flavor changing lepton decays (e.g. $\mu \rightarrow e\gamma$)
- anomalous magnetic and dipole moments



(+ Higher order corrections ...)

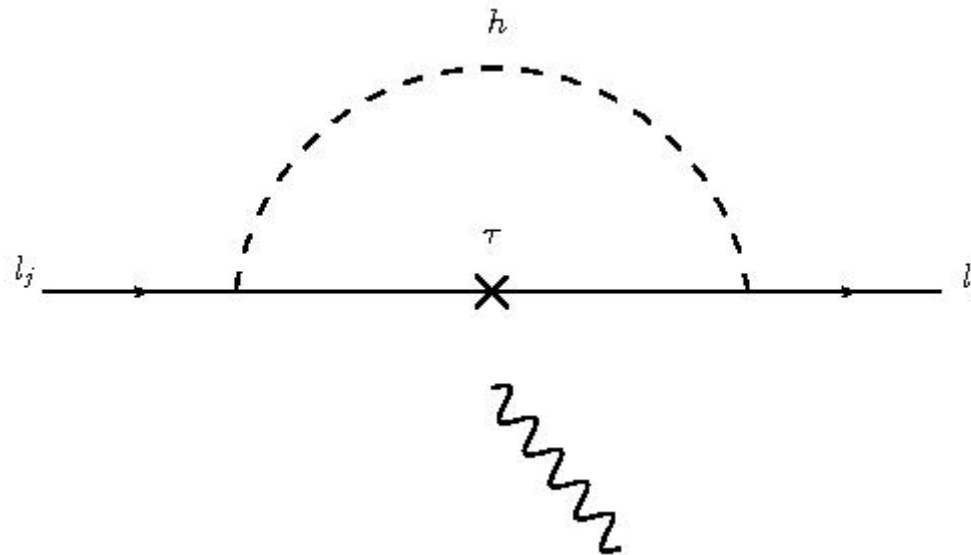
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What about them ???

(+ Higher order corrections ...)

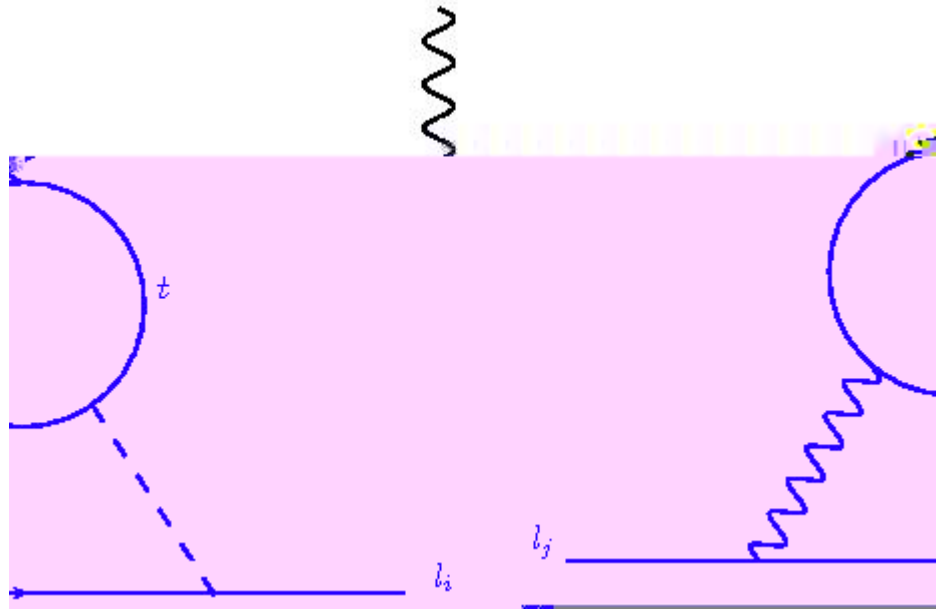
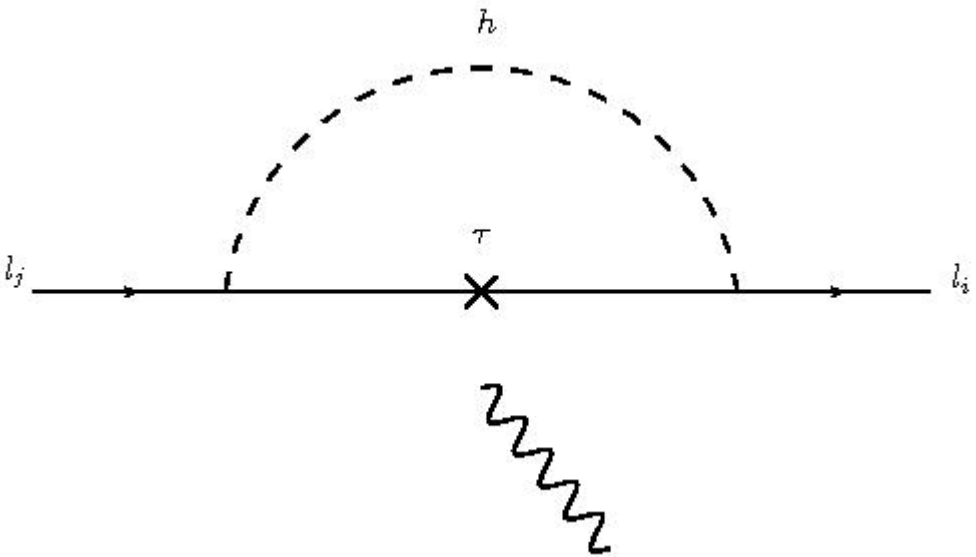
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→ One can extract constraints on the y_{ij} quantities in a **model - independent** manner!

→ Most constraining turn out to be $\mu \rightarrow e\gamma$ as well as d_e .
Some constraints also from $\mu \rightarrow eee$

Bounds on Higgs couplings : 1-loop

observable	present limit	constraint	constraint for $y_{ij} = y_{ji}$, $y_{ii} = m_i/v$
BR ($\mu \rightarrow e\gamma$)	2.	$ y_{12} + y_{21} ^{1/4} < 7 \times 10$	$\overline{ y_{ij} } < \times 1$

Bounds on Higgs couplings : 2-loops

Particularly strong constraint !

observable	constraint
BR ($\mu \rightarrow e\gamma$)	$\left(y_{12} ^2 + y_{21} ^2\right)^{1/2} < 6 \times 10^{-6}$
BR ($\tau \rightarrow \mu\gamma$)	$\left(y_{23} ^2 + y_{32} ^2\right)^{1/2} < 4 \times 10^{-2}$
BR ($\tau \rightarrow e\gamma$)	$\left(y_{13} ^2 + y_{31} ^2\right)^{1/2} < 3 \times 10^{-2}$
d_e	$ \text{Im } y_{11} < 6 \times 10^{-7}$
d_μ	$ \text{Im } y_{22} < \mathcal{O}(10^2)$
δa_e	$ \text{Re } y_{11} < 0.27$
δa_μ	$ \text{Re } y_{22} < 0.21$

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(* NB : We also checked constraints coming from $\mu \rightarrow e$ transitions in nuclei (Au, Ti) assuming SM couplings for quarks, bounds are quite weaker $\sim \mathcal{O}(10^{-3} - 10^{-4})$. *)

Dimension - 6 operators : formalism

As a first step, we do not attempt explaining the lepton mass hierarchy. We instead focus on the potential result of including only dimension - 6 operators:

$$-\Delta\mathcal{L} = H \bar{l}_{Li} e_{Rj} \left(Y_{ij}^{(0)} + Y_{ij}^{(1)} \frac{H^\dagger H}{M^2} \right) + \text{h.c.}$$

This gives us a lepton mass matrix:

$$M_{ij} = v \left(Y_{ij}^{(0)} + Y_{ij}^{(1)} \frac{v^2}{M^2} \right)$$

and a matrix of couplings of the physical Higgs boson with leptons:

$$\mathcal{Y}_{ij} = Y_{ij}^{(0)} + \boxed{3} Y_{ij}^{(1)} \frac{v^2}{M^2}$$

→ Missalignment!

The mass matrix M is diagonalized by two unitary matrices U_L, U_R :

$$U_L^\dagger M U_R = \text{diag}(m_e, m_\mu, m_\tau)$$

→ Then, the interaction matrices \mathcal{Y}_{ij} should be transformed accordingly.

Dimension - 6 operators : scan strategy

We now wish to check whether the resulting LFV couplings are viable or not. To do so:

1) We produce Yukawa textures through

$$Y = U_L \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau) U_R^\dagger$$

by scanning over the U matrices. These have the form:

$$U_L = V_L, \quad U_R = V_R \Theta, \quad \Theta = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$$

and comprise of 6 angles and 4 phases overall.

2) Then, we split the Y matrices into dim-4 and dim-6 parts, by scanning over the dim-4 matrix which is a general 3x3 complex matrix, providing an extra 9 complex parameters.

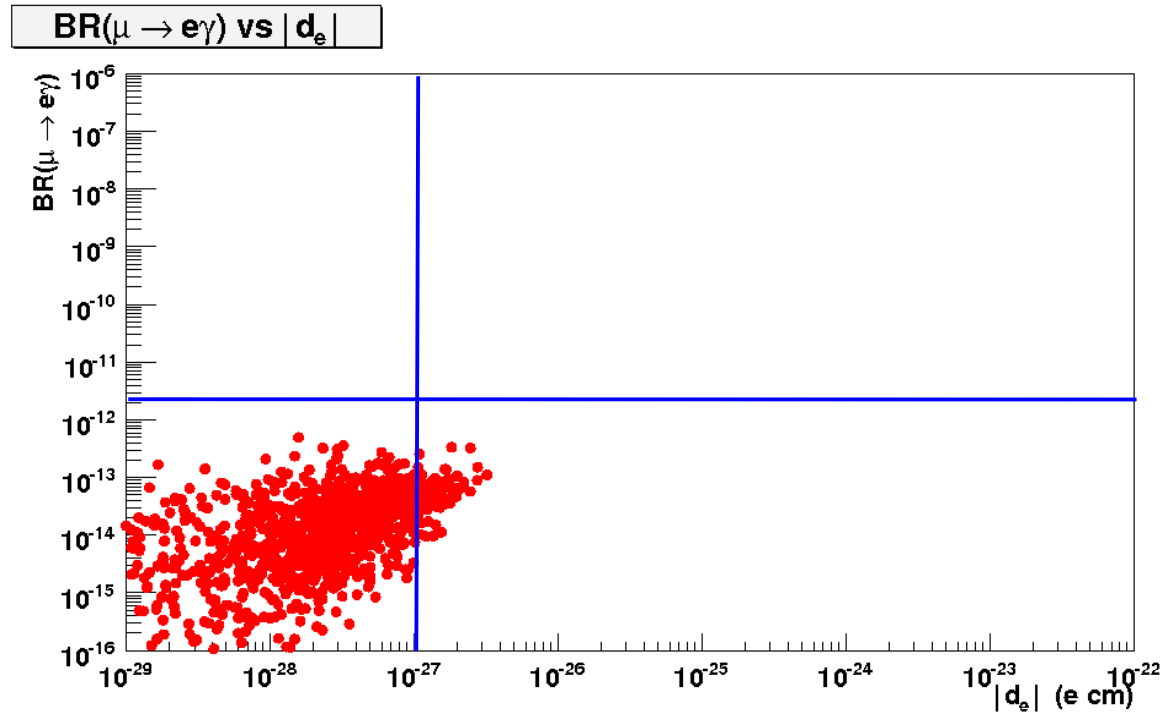
We shall be checking two things:

1) What is the relevant magnitude of the two contributions so that we get viable textures? What are the implications on the new physics scale?

2) Which are the most essential angles in the whole story?

Dimension - 6 operators @ 1 loop

Dimension - 6 operators contributing up to 10% of Yukawas

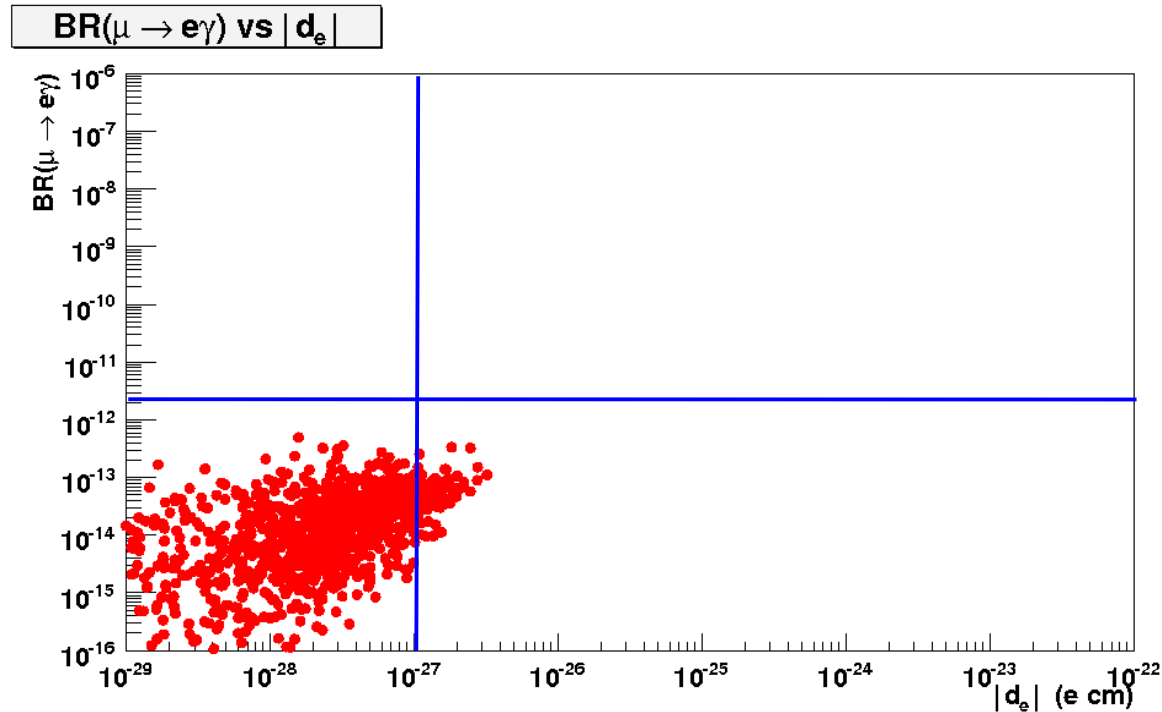


→ So, for arbitrary rotation angles the dim-6 operator contributions must be seen as rather small perturbations.

$$\left| Y_{ij}^{(1)} \frac{v^2}{M^2} \right| < 0.1 |Y_{ij}| \times \frac{200 \text{ GeV}}{m_h}$$

Dimension - 6 operators @ 1 loop

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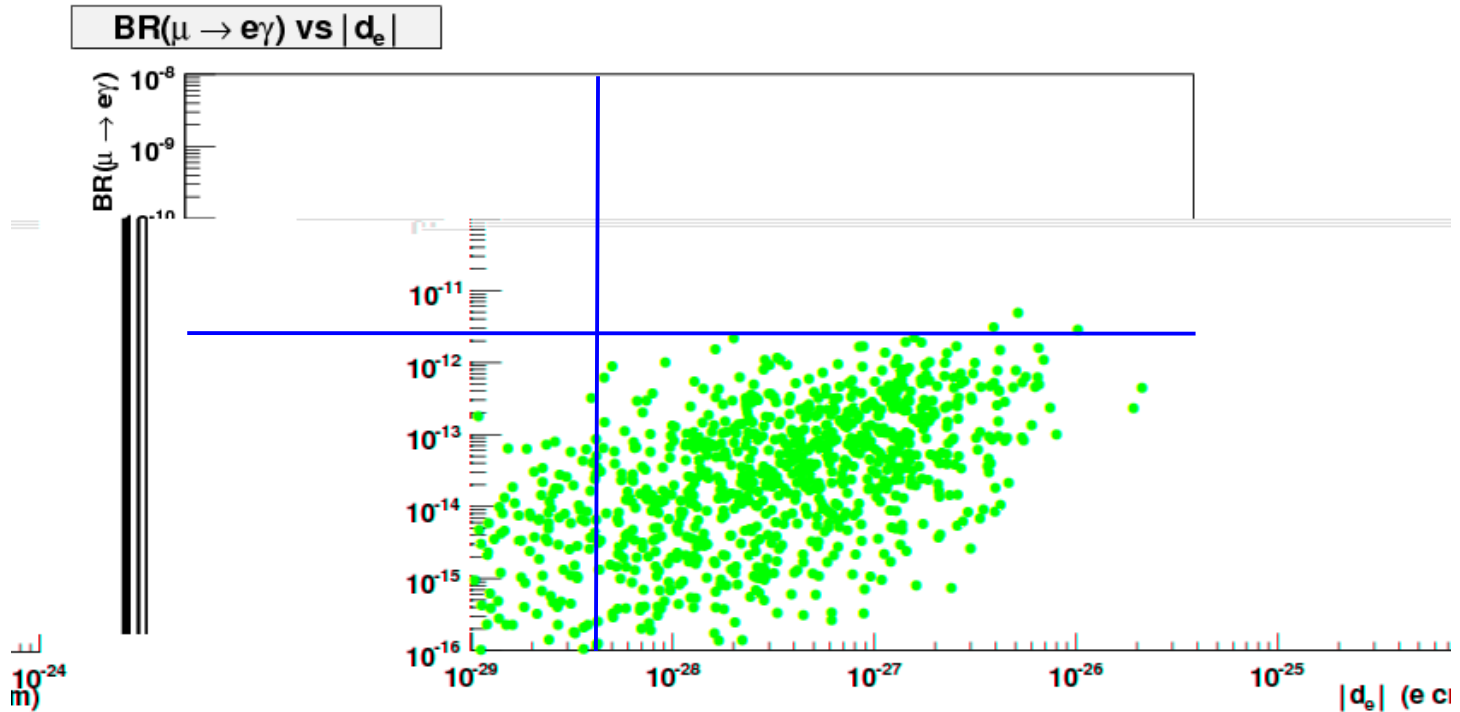
Furthermore, we find two empirical relations for the scale M for two limiting cases:

$$Y_{ij}^{(1)} \sim Y_{ij} \quad \Rightarrow \quad M > 500 \text{ GeV} \times \frac{200 \text{ GeV}}{m_h}$$

$$Y_{ij}^{(1)} \sim 1 \quad \Rightarrow \quad M > 200 \text{ TeV} \times \frac{200 \text{ GeV}}{m_h}$$

Dimension - 6 operators @ 1 loop

$$\Theta_{13}, \Theta_{23} < 0.03$$

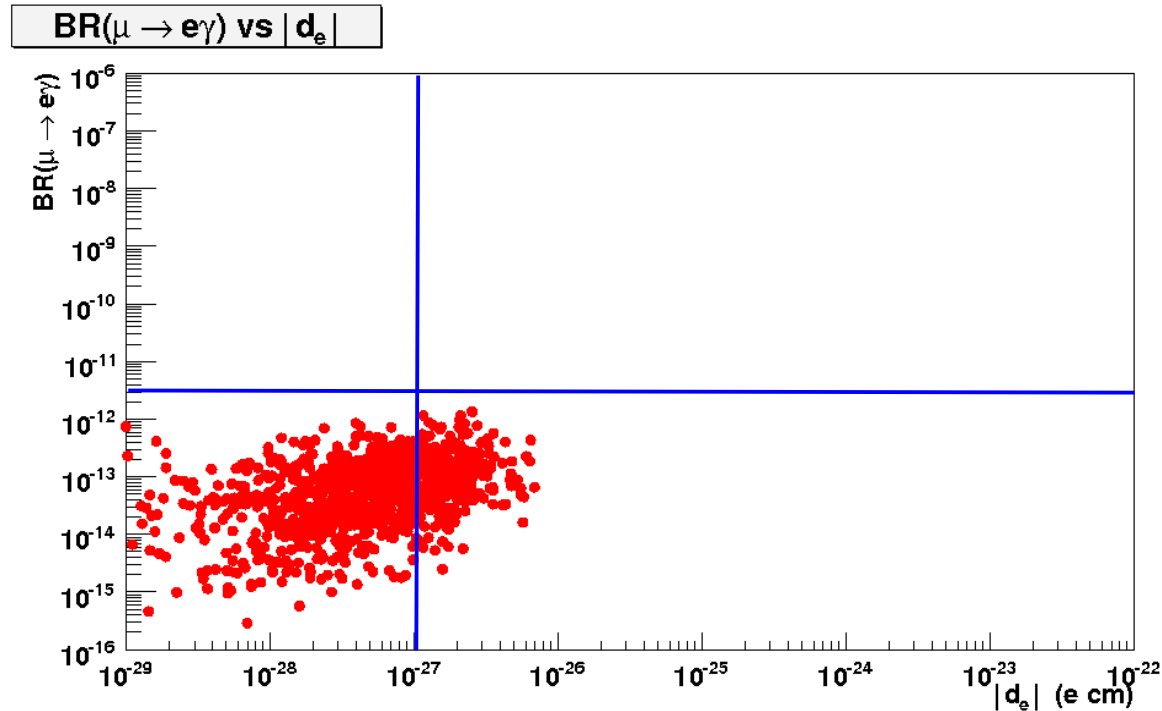


→ So, for arbitrarily large dim - 6 contributions, we can only allow for small rotation angles.

$$\theta_{13}, \theta_{23} < 3 \times 10^{-2} \times \frac{m_h}{200 \text{ GeV}}$$

Dimension - 6 operators @ 2 loops

Dimension - 6 operators contributing up to 0.1% of Yukawas

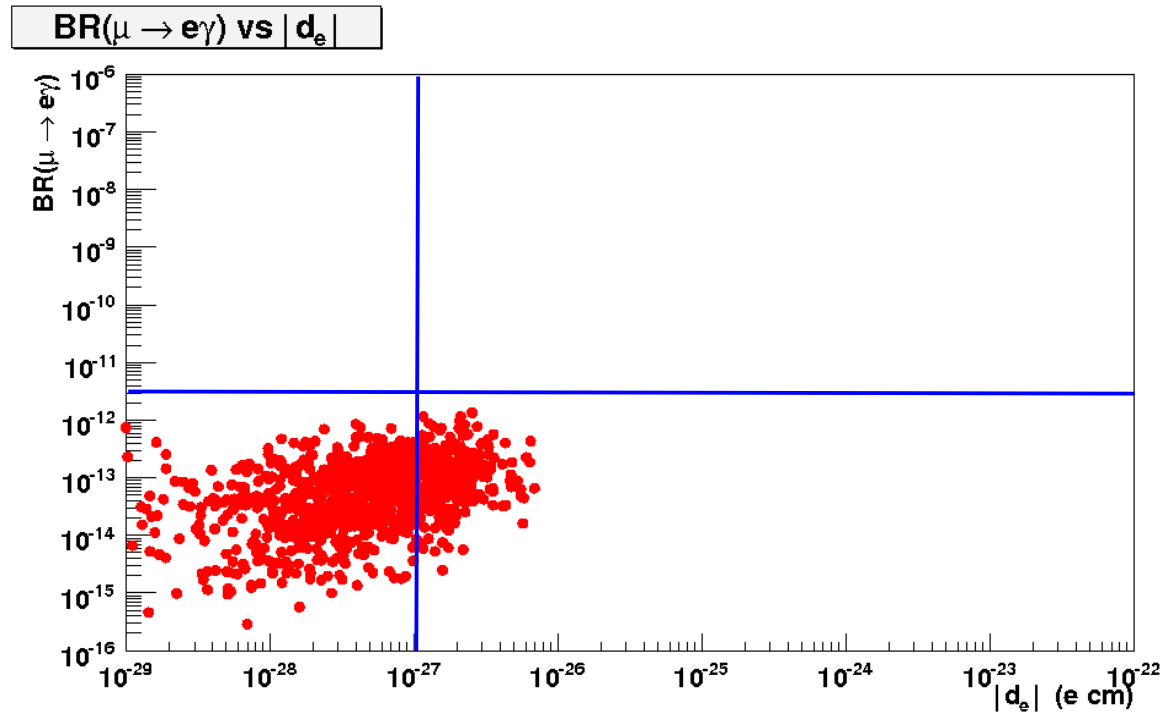


Extremely severe constraint @ 2 loops for the (1,2) - sector :

$$\left| Y_{ij}^{(1)} \frac{v^2}{M^2} \right| < 10^{-3} |Y_{ij}|$$

Dimension - 6 operators @ 2 loops

Dimension - 6 operators contributing up to 0.1% of Yukawas



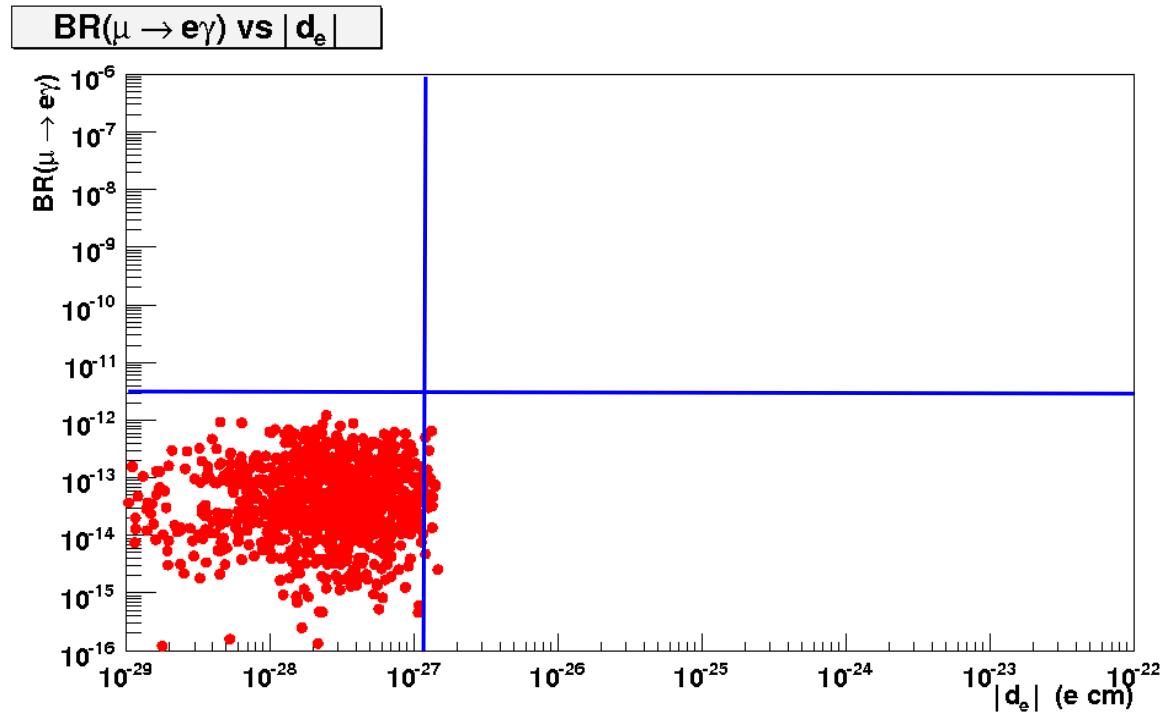
Or, seen differently :

$$Y_{ij}^{(1)} \sim Y_{ij} \quad \rightarrow \quad M > 5 \text{ TeV}$$

$$Y_{ij}^{(1)} \sim 1 \quad \rightarrow \quad M > 2000 \text{ TeV}$$

Dimension - 6 operators @ 2 loops

$$\Theta_{13}, \Theta_{23} < 10^{-1}, \Theta_{12} < 10^{-2}, \varphi' \text{'s} < 10^{-1}$$



→ The 13, 23 angles are more or less as previously, additional restrictions on the 12 mixing as well as the phases.

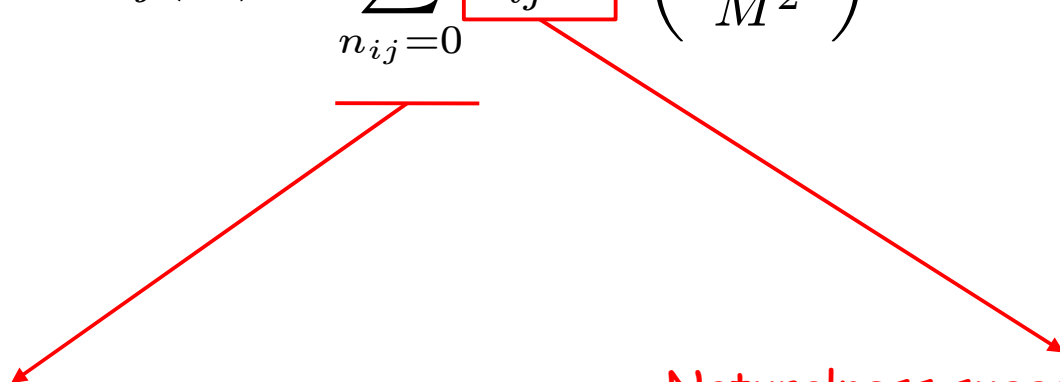
The 2 - loop contributions induce new bounds on (previously unconstrained) parameters, and actually pretty tight ones!

Higher - dimensional operators : reminder

Now, we set off to try and reproduce the lepton mass hierarchy. Reminder of the initial hypothesis :

$$\mathcal{L}_Y = Y_{ij}^u(H) Q_{Li} u_{Rj} H - Y_{ij}^d(H) Q_{Li} d_{Rj} H^c - Y_{ij}^l(H) L_{Li} e_{Rj} H^c + \text{h.c.}$$

We consider the general expansion of the yukawas:

$$Y_{ij}(H) = \sum_{n_{ij}=0}^{\infty} \boxed{\kappa_{ij}^{(n_{ij})}} \left(\frac{H^\dagger H}{M^2} \right)^{n_{ij}}$$


- In principle an infinite series, where the lowest order terms should dominate for each Yukawa coupling.

- Naturalness suggest that these coefficients should be $O(1)$.
- They could also be zero, if for instance some symmetry is at play in the high-energy theory.

Higher - dimensional operators : formalism

The idea is to indeed consider that the series' coefficients vanish up to some **generation - dependent order n** :

$$Y_{ij}(H) = \sum_{n_{ij}=0}^{\infty} \kappa_{ij}^{(n_{ij})} \left(\frac{H^\dagger H}{M^2} \right)^{n_{ij}}$$

$O(1)$, vanishing up to some generation - dependent order

→ Then, the mass hierarchy is generated by : $\varepsilon = v^2/M^2 \ll 1$ (by $m_b/m_t \rightarrow \varepsilon \sim 1/60$).

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The Yukawa lepton textures then take the form :

$$Y_{ij} = c_{ij} \epsilon^{n_{ij}}, \quad n_{ij} = \text{round}(\log_\epsilon |Y_{ij}|)$$

But remember that

$$Y = U_L \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau) U_R^\dagger$$

which also fixes the $O(1)$ coefficients c_{ij} .

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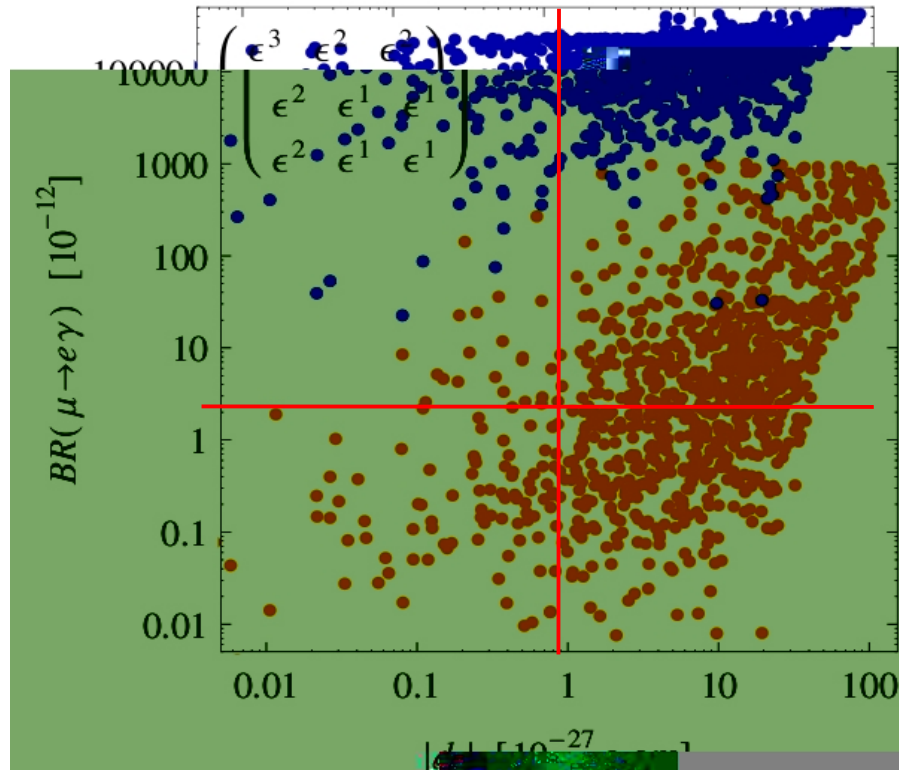
Eventually, rotating to the mass eigenstate basis we can obtain the couplings :

$$y_{ij} = (U_L)_{ki}^* (2n_{kl} + 1) Y_{kl} (U_R)_{lj}$$

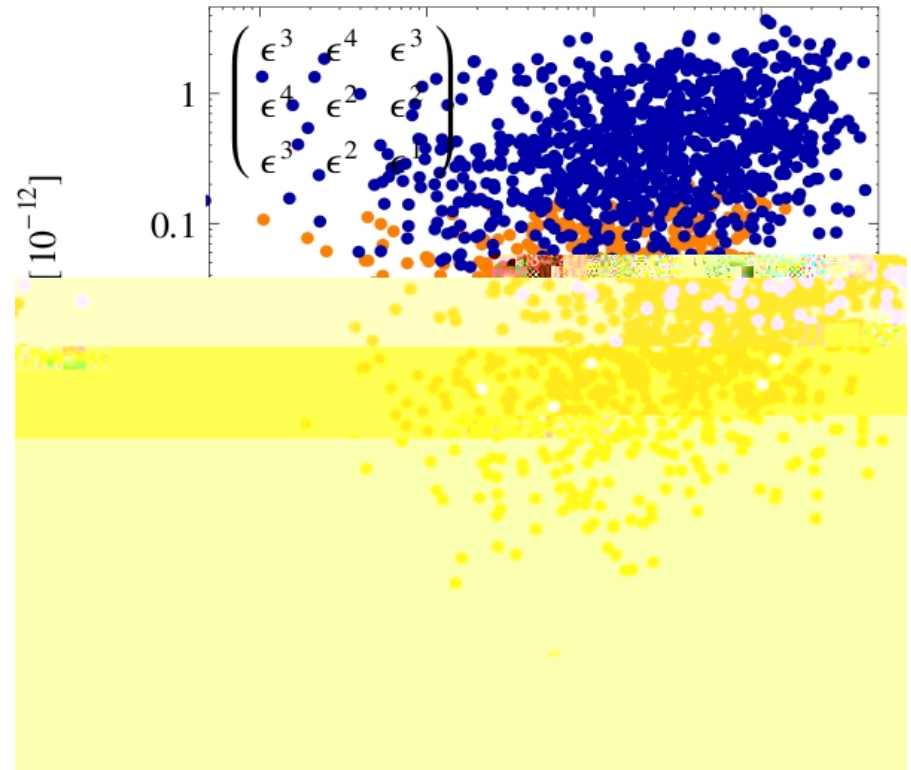
→ A texture will be characterized as "factorizable" if $n_{ij} = a_i + b_j$.
The Froggatt - Nielsen mechanism implies factorizable textures.

Higher - dimensional operators : some results

Factorizable



Non - factorizable

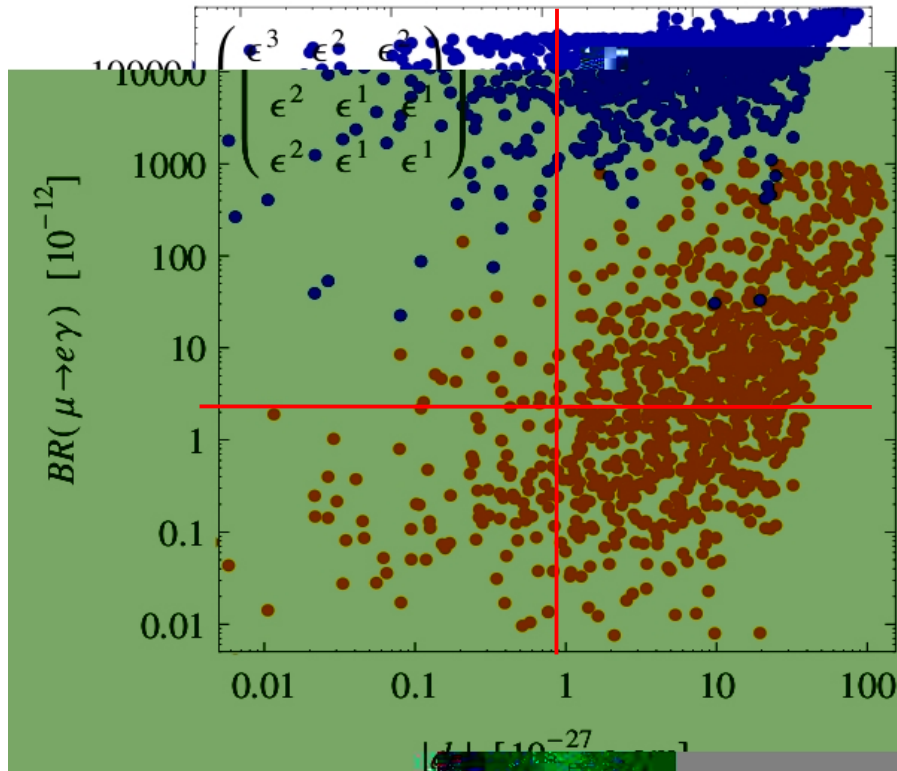


Remarks :

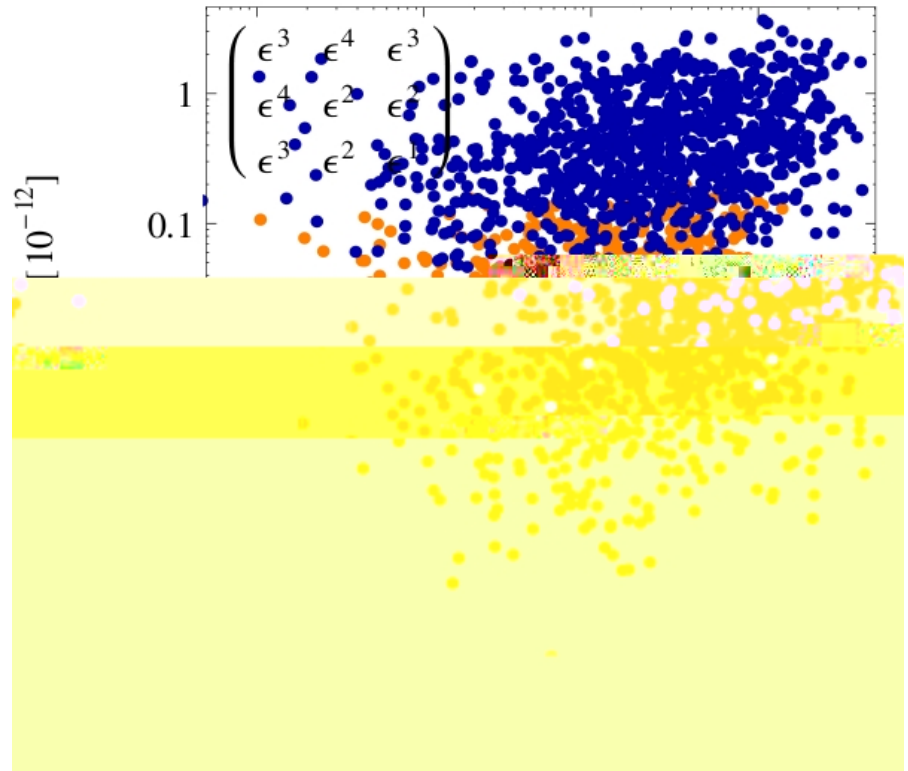
- A 3-2-1 structure on the main diagonal seems natural for the e- μ - τ mass hierarchy.
- The F-N mechanism kind of restricts textures. Especially if the 3-2-1 pattern is kept on the main diagonal, the choices are rather limited...

Higher - dimensional operators : results

Factorizable



Non - factorizable



→ Textures with strong hierarchies (\leftrightarrow small intergenerational mixing) are strongly preferred!

Conclusions

- The framework of HDYC can explain the mass hierarchy of the fermion sector, which in the Standard Model finds no justification.
- This assumption can result to important phenomenological consequences in Higgs physics, CP - violation as well as Lepton Flavor Violation observables and EDMs.
- We saw that it is possible to extract model-independent constraints on the couplings of the physical higgs boson with leptons from LFV and EDM/AMM observables. Of particular strength can be 2-loop contributions!
- As a first step, we examined the impact of dimension-6 operators on lepton sector observables, finding that :
 - 1) If we do not restrict the rotation angles, small contributions from these operators are strongly favored.
 - 2) If we allow for significant contributions from dimension-6 operators, then small rotation angles are strongly preferred.
- Then, we assumed that n-dimensional contributions vanish up to a generation-dependent order n. In this way we can reproduce the lepton mass pattern. We saw that :
 - 1) "Factorizable" (Froggatt-Nielsen - inspired) Yukawa textures can be restrictive.
 - 2) Hierarchical textures, implying small intergenerational mixing, are preferred due to the strong constraints on LFV observables.

Merci !

Relevant LFV and EDM Lagrangians

$$\Delta\mathcal{L} = -\frac{y_{ij}}{\sqrt{2}} h \bar{l}_i P_R l_j + \text{h.c.}$$

$$\mathcal{L}_{\text{eff}_1} = e L_{ij} \bar{l}_i \sigma^{\mu\nu} F_{\mu\nu} P_L l_j + \text{h.c.}$$

$$\mathcal{L}_{\text{eff}_2} = e \text{Re}L_{ii} \bar{l}_i \sigma^{\mu\nu} F_{\mu\nu} l_i - ie \text{Im}L_{ii} \bar{l}_i \sigma^{\mu\nu} F_{\mu\nu} \gamma_5 l_i + \text{h.c.}$$

Where :

$$L_{ij} = \frac{y_{3i}^* y_{j3}^*}{64\pi^2 m_h^2} m_\tau \ln \frac{m_\tau^2}{m_h^2}$$

Observables @ 1-loop

$$\Delta\mathcal{L} = -\frac{y_{ij}}{\sqrt{2}} h \bar{l}_i P_R l_j + \text{h.c.}$$

$$\text{BR}(l_j \rightarrow l_i \gamma) = \text{BR}(l_j \rightarrow l_i \nu_j \bar{\nu}_i) \times \frac{192\pi^3 \alpha}{G_F^2 m_j^2} (|L_{ij}|^2 + |L_{ji}|^2)$$

$$\text{BR}(l_j \rightarrow l_i l_k l_k^+) = \text{BR}(l_j \rightarrow l_i \nu_j \bar{\nu}_i) \times \frac{(4 - \delta_{ik})}{256 G_F^2 m_h^4} |y_{kk}|^2 (|y_{ij}|^2 + |y_{ji}|^2)$$

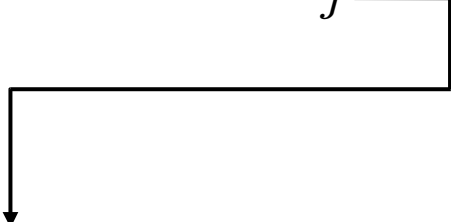
$$\begin{aligned} |\delta a_\mu| &= 4m_\mu |\text{Re}L_{22}|, \\ |d_i| &= 2e |\text{Im}L_{ii}| \end{aligned}$$

$$L_{ij} = \frac{y_{3i}^* y_{j3}^*}{64\pi^2 m_h^2} m_\tau \ln \frac{m_\tau^2}{m_h^2}$$

Observables @ 2-loops

$$\Delta\mathcal{L} = -\frac{y_{ij}}{\sqrt{2}} h \bar{l}_i P_R l_j + \text{h.c.}$$

$$\text{BR}(l_j \rightarrow l_i \gamma) = \text{BR}(l_j \rightarrow l_i \nu_j \bar{\nu}_i) \times \frac{8\alpha^3 v^2}{3\pi^3 m_j^2} f^2(z) (|y_{ij}|^2 + |y_{ji}|^2)$$

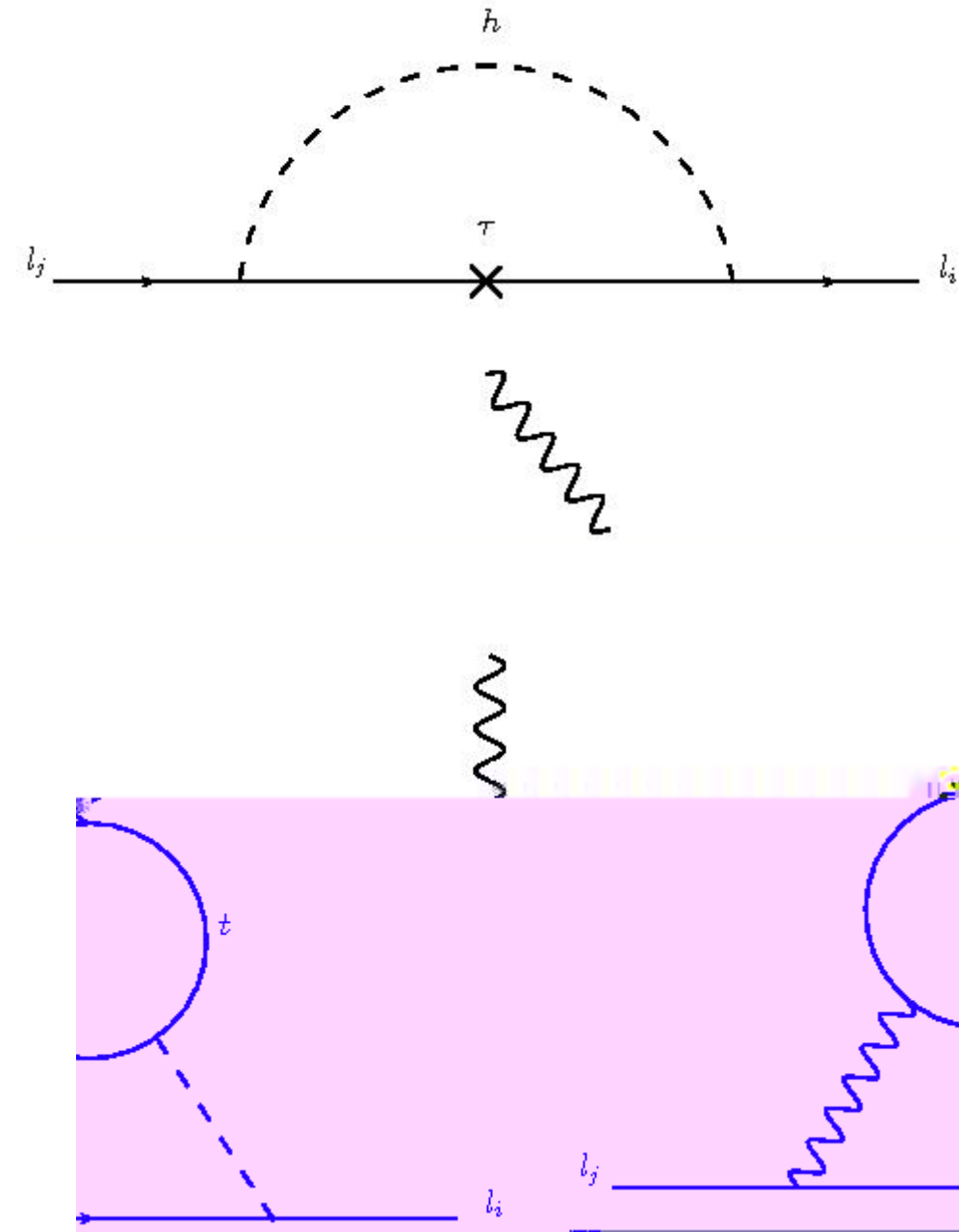

$$f(z) = \frac{1}{2} z \int_0^1 dx \frac{1 - 2x(1-x)}{x(1-x) - z} \ln \frac{x(1-x)}{z}$$

$$|\delta a_\mu| = 4m_\mu |\text{Re}L_{22}|,$$

$$|d_i| = 2e |\text{Im}L_{ii}|$$

$$L_{ii} = \frac{\alpha}{24v\pi^3} f(z) y_{ii}^*$$

The Barr - Zee contributions



This contribution has :

- 1 loop suppression factor
- 1 gauge coupling (not so small)
- 1 helicity suppression
- 2 lepton Yukawas (VERY small !!!)

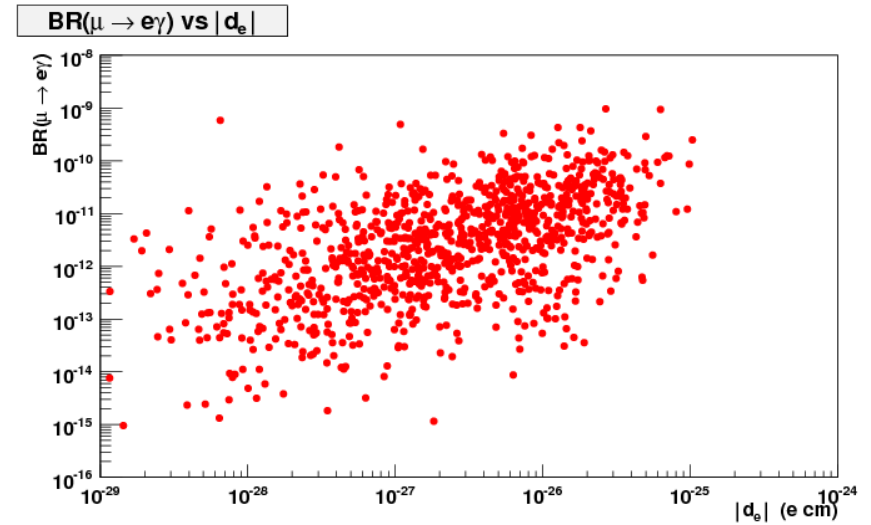
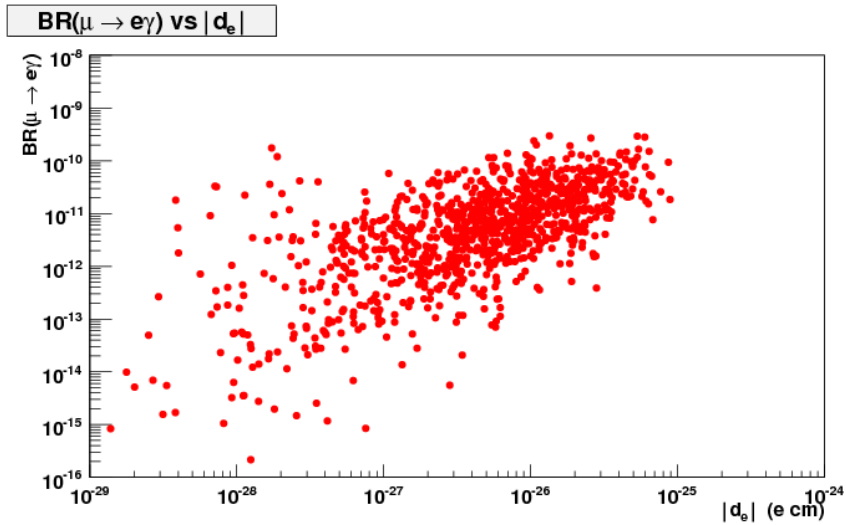
This contribution has :

- 2 loop suppression factors
- One t-quark Yukawa ($O(1)$)
- 3 gauge couplings (not so small)
- No helicity suppression
- 1 lepton Yukawa

More examples @ 1 loop

Dim-6 contributing up to
50% of the mass matrix

$$\Theta_{13}, \Theta_{23} < 0.1$$



Inclusion of the 2-loop contributions completely kills all viable models !

Comparison to Froggatt - Nielsen mechanism :

Q : Doesn't this thing remind a lot of the Froggatt - Nielsen mechanism ?

A : Yes, but there are three important differences:

- 1) The NP scale M is, in our setup, around the TeV scale unlike in the F-N mechanism where it is usually associated with some really high scale.
- 2) The expansion parameter is quite smaller (here chosen $O(1/60)$) than in F-N constructions (typical $O(0.22)$).
- 3) There is no "flavon" field : H^+H doesn't carry quantum numbers.
(different situation in SUSY models, see discussion in arXiv:0804.1753)