Glueballs:

At the interface between lattice QCD and constituent models

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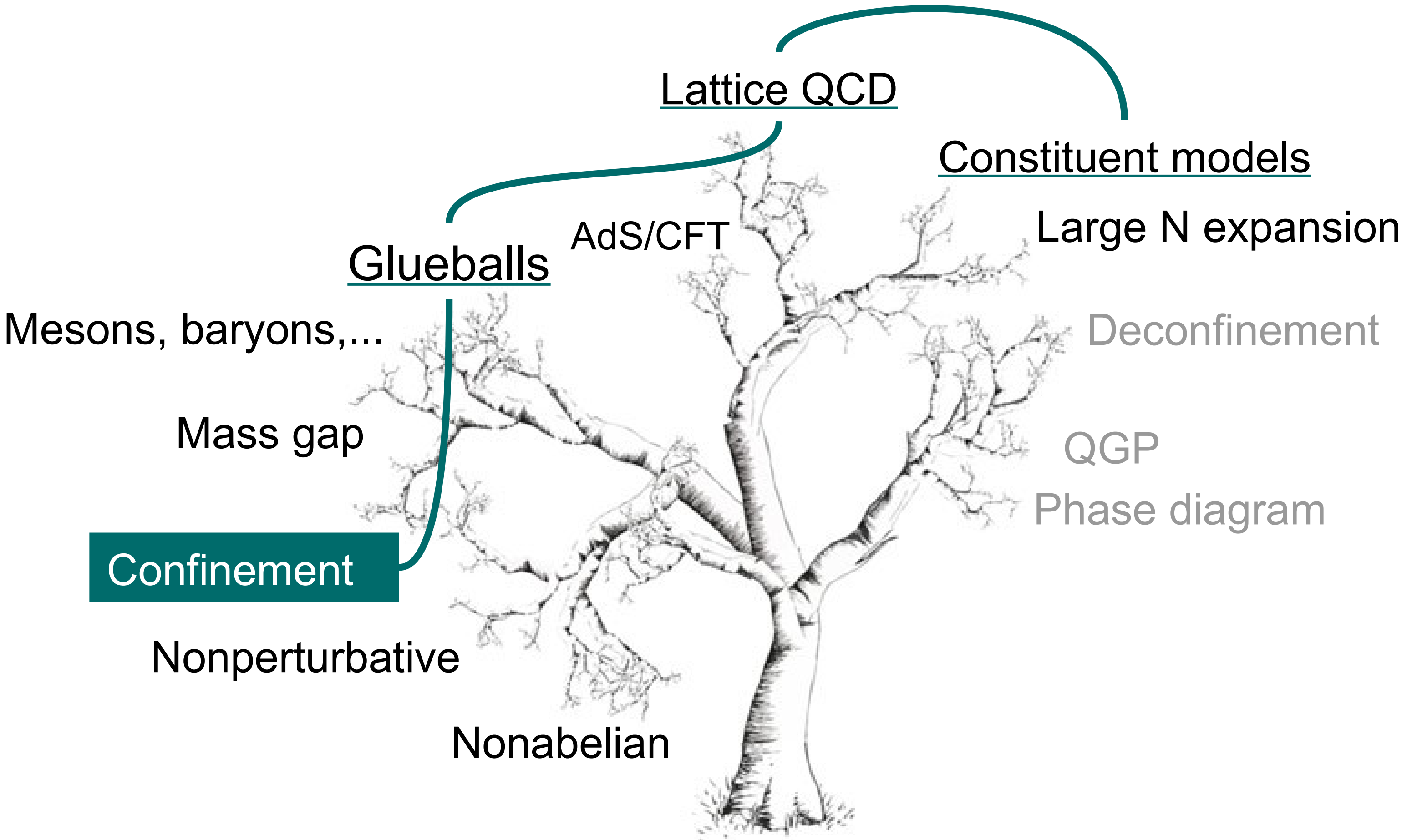
Prologue



$$\mathcal{L}_{QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu + m_f)\psi_f$$

Outline

A summary about glueballs

- Experimental data
- Lattice QCD
  - Mass spectrum
  - Wave function
- Effective approaches

How to build a constituent approach?

- Why?
- Informations from the lattice
  - Constituent gluons
  - Potential

Glueball mass spectrum

- Two- and three-body states
- Large N limit

Thermodynamics

Conclusions

A summary about glueballs

Experimental data

Nothing unambiguous yet

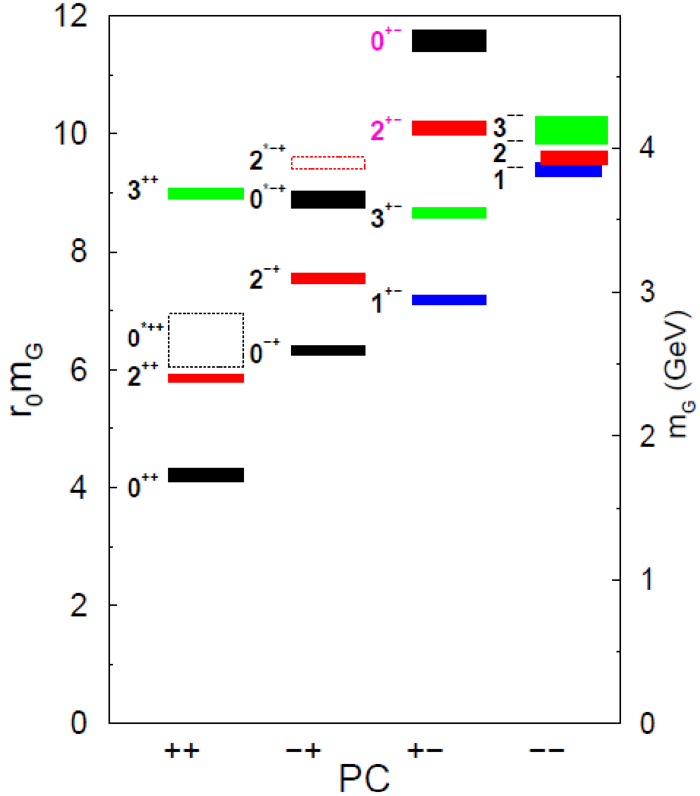
- Too many $0(0^{++})$ states for the quark model
  - PDG: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$
  - Mixed states involving: $|u\bar{u}\rangle + |d\bar{d}\rangle,\ |s\bar{s}\rangle,\ |G\rangle$
- Unclear status of the $0(0^{-+})$ state $\eta(1405)$
- Future: PANDA, GlueX, ...

Lack of unquenched QCD results

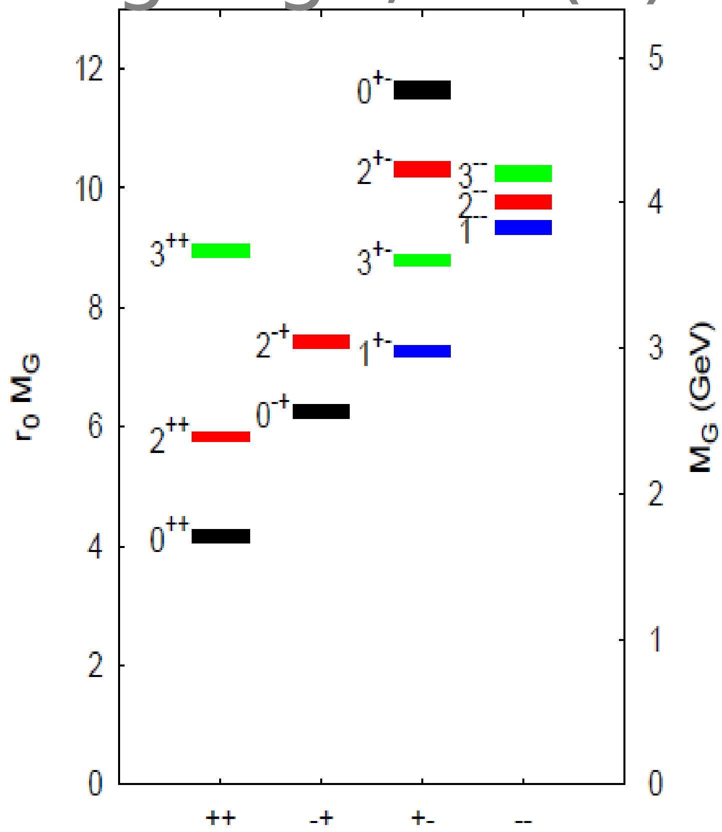
- Mixings in Fock space
- Decay widths

Lattice results (I)

Mass spectrum: Pure gauge, SU(3)



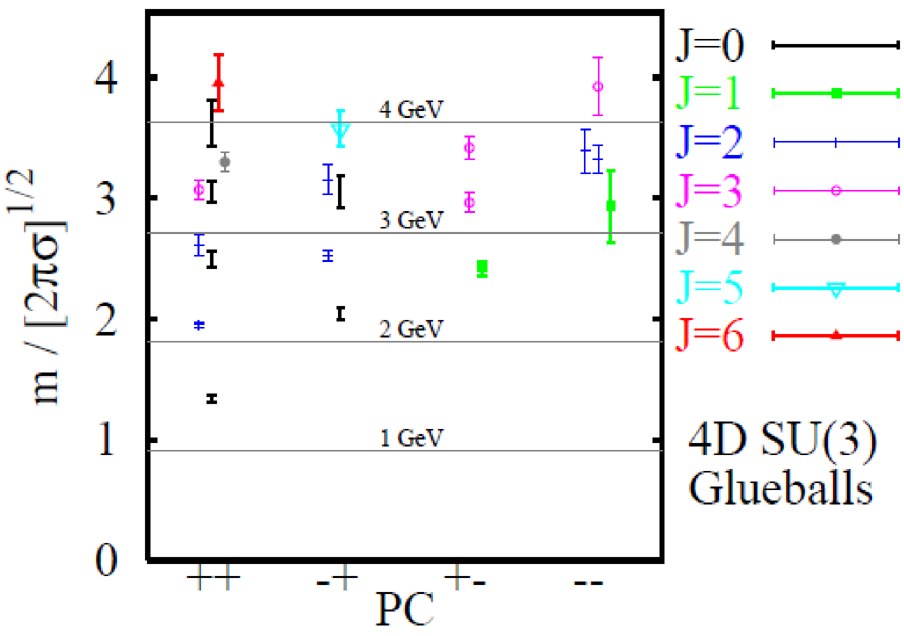
C. Morningstar and M. Peardon,
Phys. Rev. D **60,** 034509 (1999)

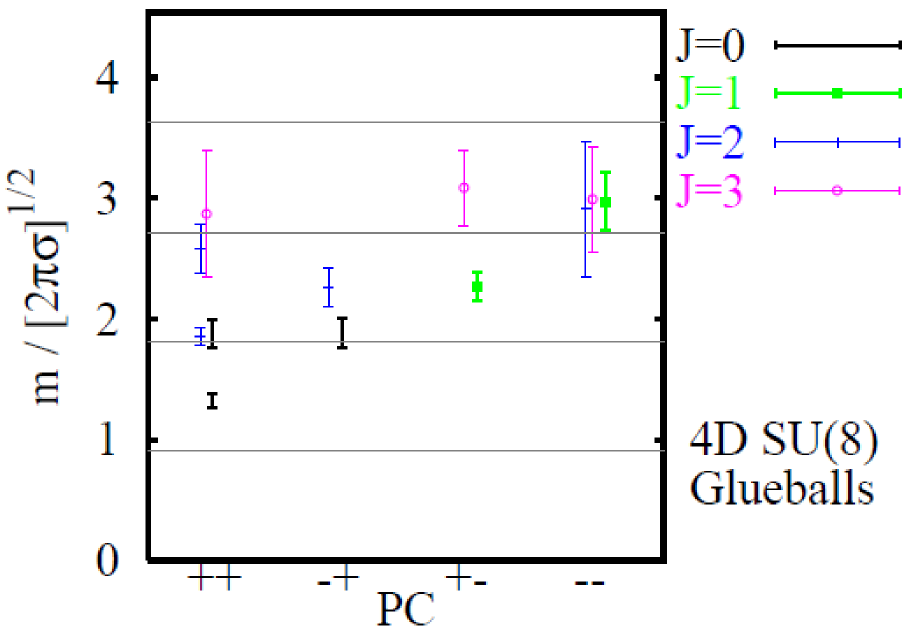


Y. Chen *et al.*,
Phys. Rev.D **73**, 014516 (2006)

Lattice results (II)

Pure gauge, SU(3) and SU(8)

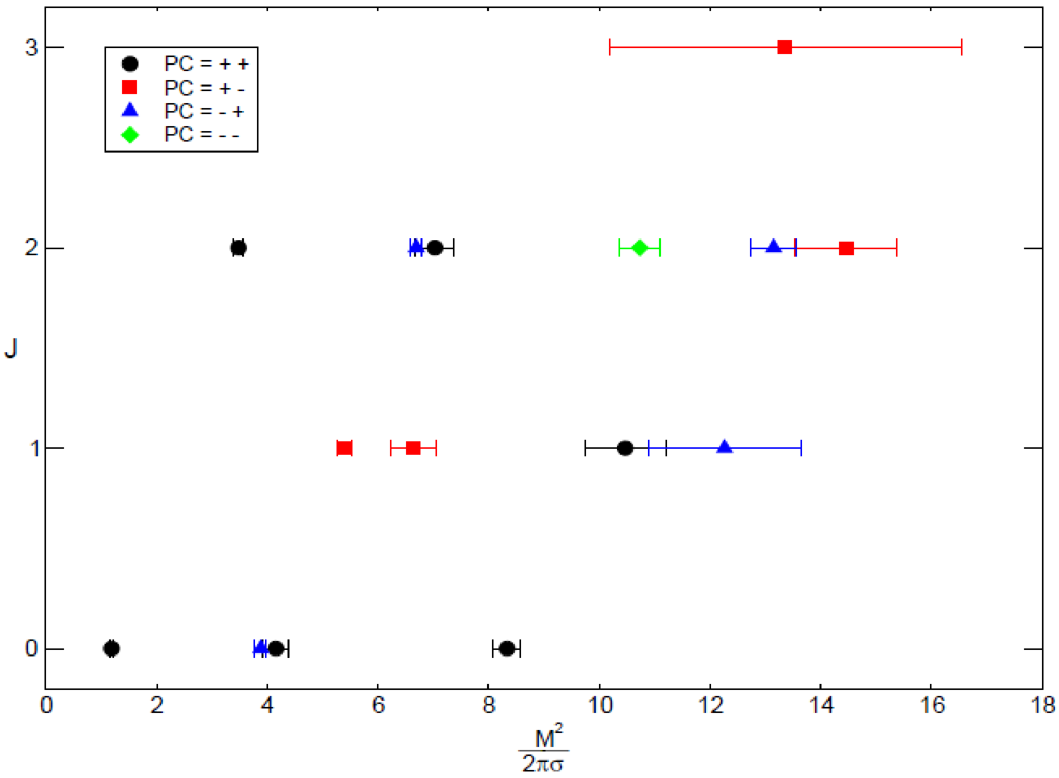




H. B. Meyer and M. J. Teper, Phys. Lett. B **605**, 344 (2005)

Lattice results (III)

Large N limit



B. Lucini, A. Rago, and E. Rinaldi, JHEP **1008**, 119 (2010)

Lattice results (IV)

Structure of the spectrum

- Lightest states with C = + : 0^{++}, 2^{++}, 0^{-+}
  - Scalar always the lowest-lying, 1400-1800 MeV
  - Range of some f_0 states
- No light 1^{P+} state

Large N limit

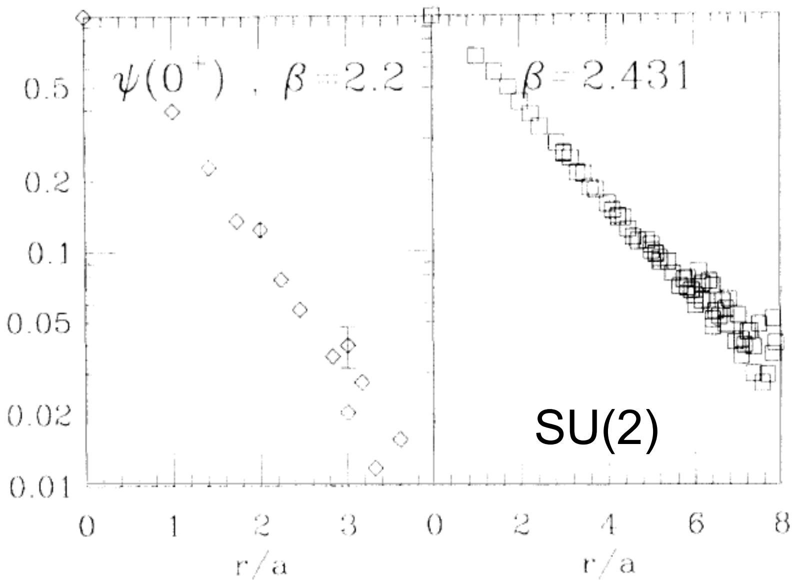
- Behavior in $$M_G(N) = M_G(\infty) + \frac{\theta}{N^2}$$
- θ compatible with 0

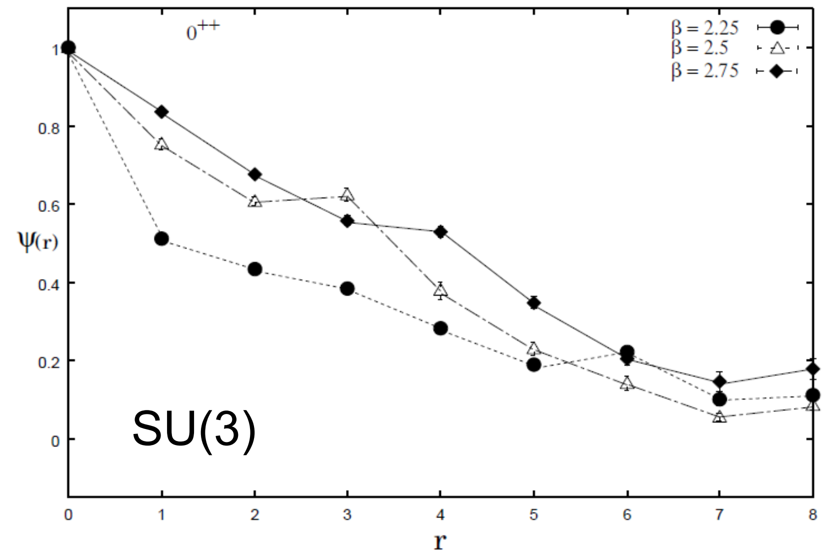
Lattice results (V)

0^{++} Wave functions

- Bethe-Salpeter for a two-gluon state

$$\chi(\vec{r}) = \langle 0| s^{\mu\nu} \int d\hat{r}\, Y_{lm}(\hat{r})\, A^{\dagger}_{\mu}(\vec{x}) A_{\nu}(\vec{x}+\vec{r})|G\rangle$$





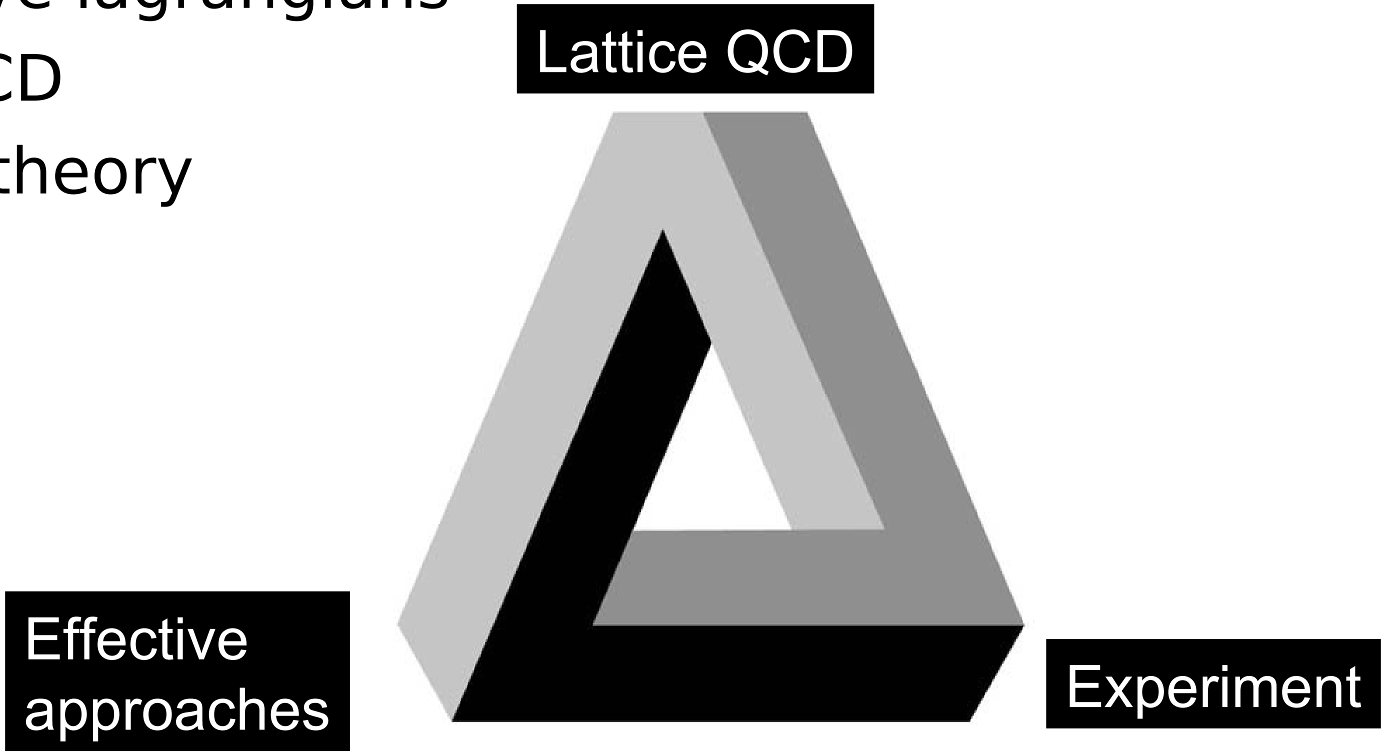
P. de Forcrand and K. F. Liu,
Phys. Rev. Lett. **69**, 245 (1992)

M. Loan and Y. Ying,
Prog. Th. Phys. **116**, 169 (2006)

Effective approaches

Large amount of theoretical works

- Potential models / Constituent approaches
- Effective lagrangians
- AdS/QCD
- String theory
- ...



How to build a constituent approach?

Why « constituent » ? (I)

Correlators $\left\langle \Theta_G^\dagger(\tau)\Theta_G(0) \right\rangle \propto \mathrm{e}^{-m_G \tau}$

- Glueball operators
  - $0^{++} = \vec{E}_a^2 \pm \vec{B}_a^2$
  - $1^{+-} = d_{abc}(\vec{E}_a \cdot \vec{E}_b)\vec{B}_c$
- Gluelumps
  - $1^{+-} = \vec{B}_a$

2 gluons

3 gluons ?

1 gluons

Large N

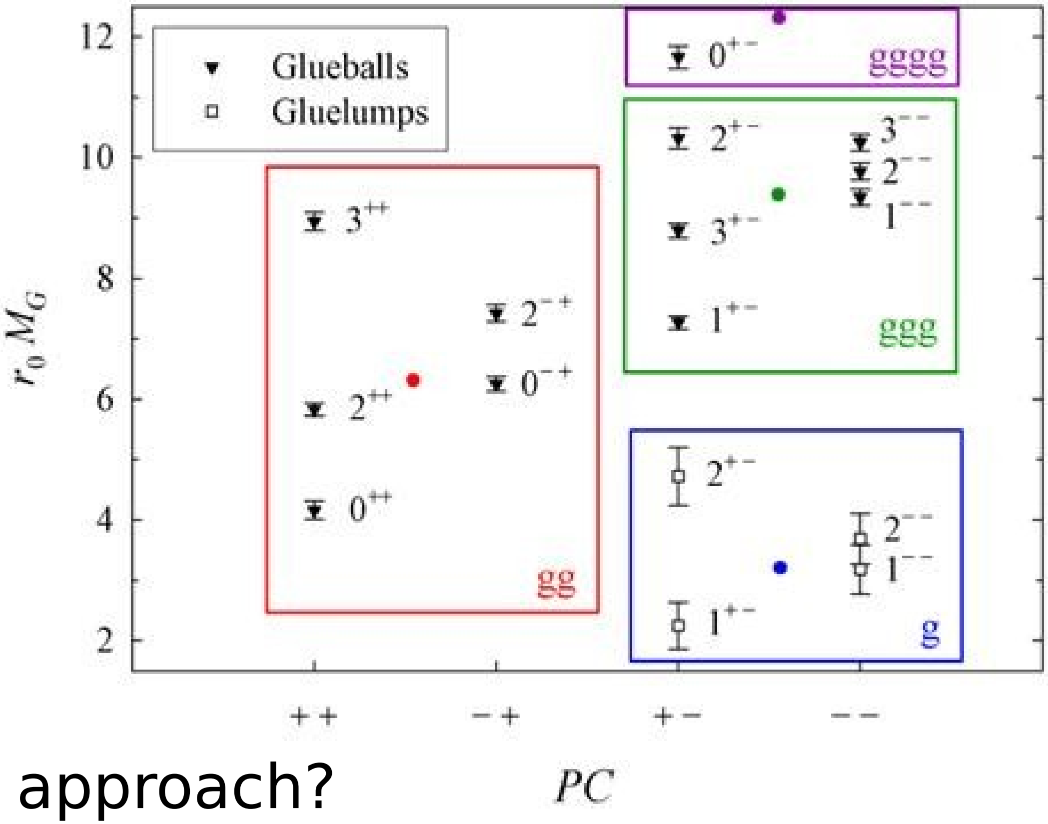
- Constituent approach as good (or bad) for N = 3 than for large N

Mass gap

N. Boulanger, FB, V. Mathieu and C. Semay, Eur. Phys. J. A **38**, 317 (2008)

Why « constituent » ? (II)

Assumption: Glueball = bound state of gluons



- Hamiltonian approach?

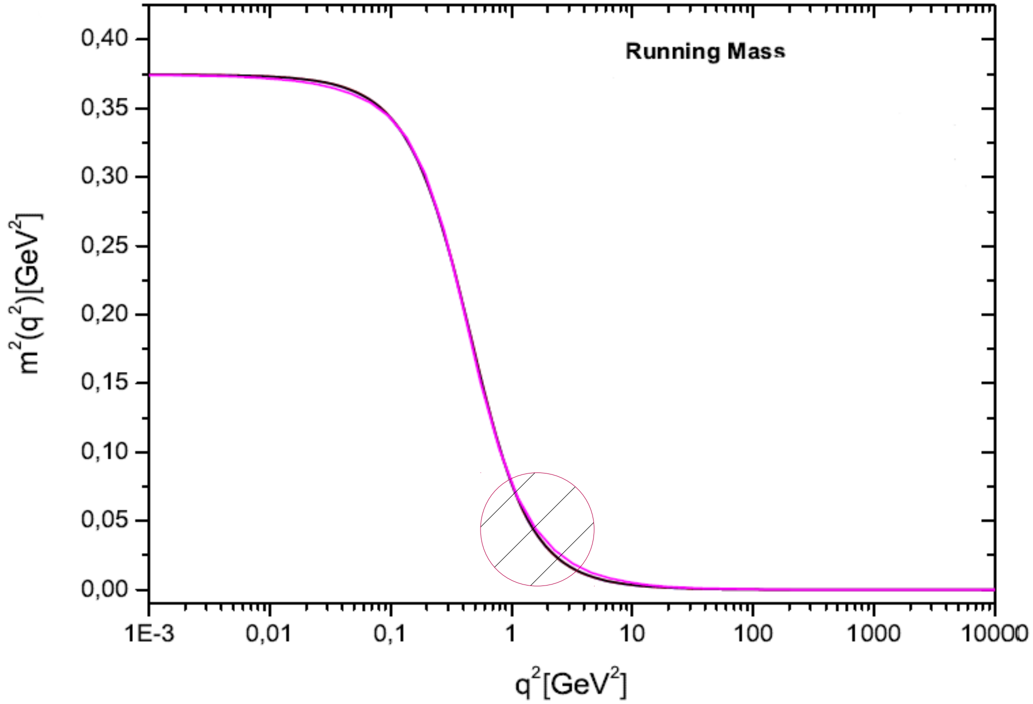
Gluon's features (I)

Color octet

- Singlet if more than 2 gluons
- Charge conjugation $\hat{C}\, A_\mu\, \hat{C}^{-1} = -A_\mu^T$
- Glueball's *C*

Gluon mass

- 0 bare mass
- Generated
  - About 600 MeV at $q^2=0$
  - Quite small above $q^2= 1$ GeV



A.C. Aguilar and J. Papavassiliou, Phys. Rev. D **81**, 034003 (2010)

Gluon's features (II)

Spin degree of freedom

- Early works: spin 1, $S_z = -1, 0, +1$
  - Usual LS basis like quark models
  - Too many states when compared to lattice

V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E **18**, 1 (2009)

- Our approach: transverse gluons
  - Zero mass
  - Helicity 1, $\lambda = \pm 1$
  - Jacob and Wick's helicity formalism
  - Only the lattice states

V. Mathieu, FB and C. Semay, Phys. Rev. D **77**, 114022 (2008).
M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959).

Helicity formalism (I)

Two-gluon states

$$\left|\lambda_1, \lambda_2; J^P, M, \epsilon\right\rangle = \frac{1}{\sqrt{2}} \left\{ \Omega^J_{M,\lambda_1-\lambda_2} \left[|\psi(\vec{p}, \lambda_1)\rangle \otimes |\psi(-\vec{p}, \lambda_2)\rangle \right] + \epsilon\, \Omega^J_{M,\lambda_2-\lambda_1} \left[|\psi(\vec{p}, -\lambda_1)\rangle \otimes |\psi(-\vec{p}, -\lambda_2)\rangle \right] \right\}$$

$$\Omega^J_{M,\lambda}[X] = \left[\frac{2J+1}{4\pi}\right]^{1/2} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, \sin\theta \times \mathcal{D}^{J*}_{M,\lambda}(\phi, \theta, -\phi)\, R(\phi, \theta, -\phi)\, X(\phi, \theta)$$

Quantum numbers

$$J \geq |\lambda_1 - \lambda_2|$$

$$P = \epsilon(-)^J, \quad C = +$$

Helicity formalism (II)

Helicity states + Pauli principle

- Color symmetric, spin-space symmetric
- No 1^{++} and 1^{-+} states
  - Yang's theorem, no $\rho \to \gamma\gamma$
  - Lattice, no light J = 1 glueball
- No 3^{-+}, 5^{-+}, 7^{-+}, ...
- Matrix elements $\left\langle \vec{L}^{\,2} \right\rangle = J(J+1) + 2\lambda_1\lambda_2$
- Examples

$$\left|0^{++}\right\rangle = \sqrt{\frac{2}{3}}\left|L=0, S=0\right\rangle + \sqrt{\frac{1}{3}}\left|L=2, S=2\right\rangle$$

$$\left|0^{-+}\right\rangle = -\left|L=1, S=1\right\rangle$$

Interaction potential (I)

Hamiltonian: ansatz $H_{gg} = 2\sqrt{\vec{p}^2} + V(r)$

- 0^{++} Mass and wave function from the lattice
- Inverse problem

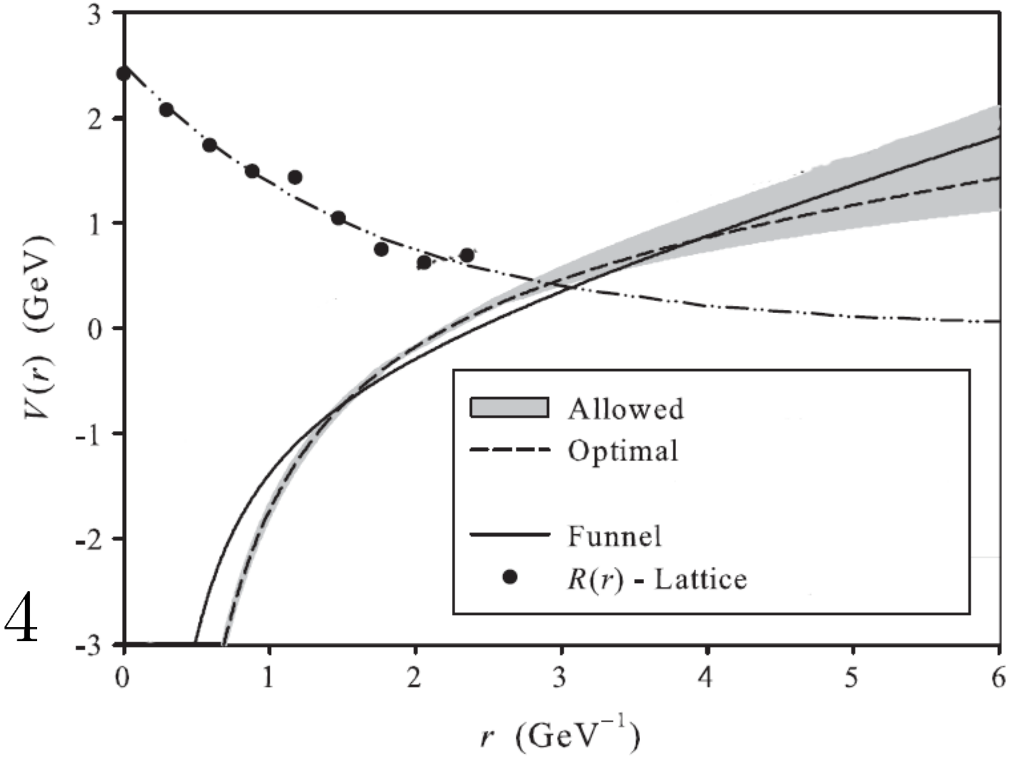
Funnel-like

- Standard,
  - Casimir scaling

$$V(r) = \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r} - D$$

$$\sigma = 0.185\ \mathrm{GeV}^2,\ \alpha_s = 0.4$$

$$D = 0.45\,\mathrm{GeV}$$



FB, Phys. Rev. D **79**, 037503 (2009)

Interaction potential (II)

Instanton-induced forces

- Suggestion
  - Attractive in the scalar channel
  - Repulsive (same magnitude) in the pseudoscalar one

  H. Forkel, Phys. Rev. D **71**, 054008 (2005)

- Negative D-constant

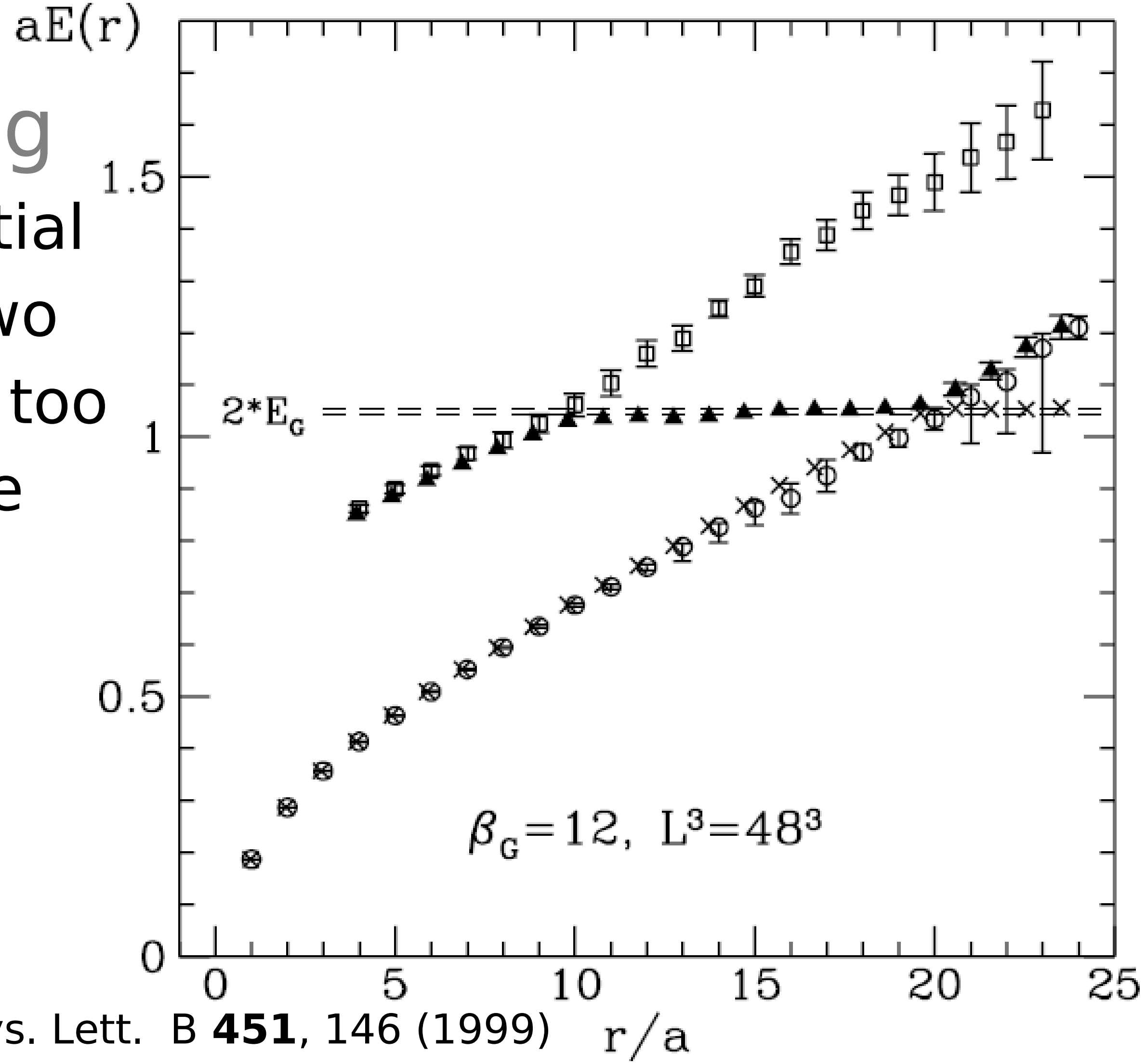
Spin-effects

- Neglected in first approximation

Interaction potential (III)

Adjoint string

- Funnel potential
- Creation of two gluelumps at too large distance

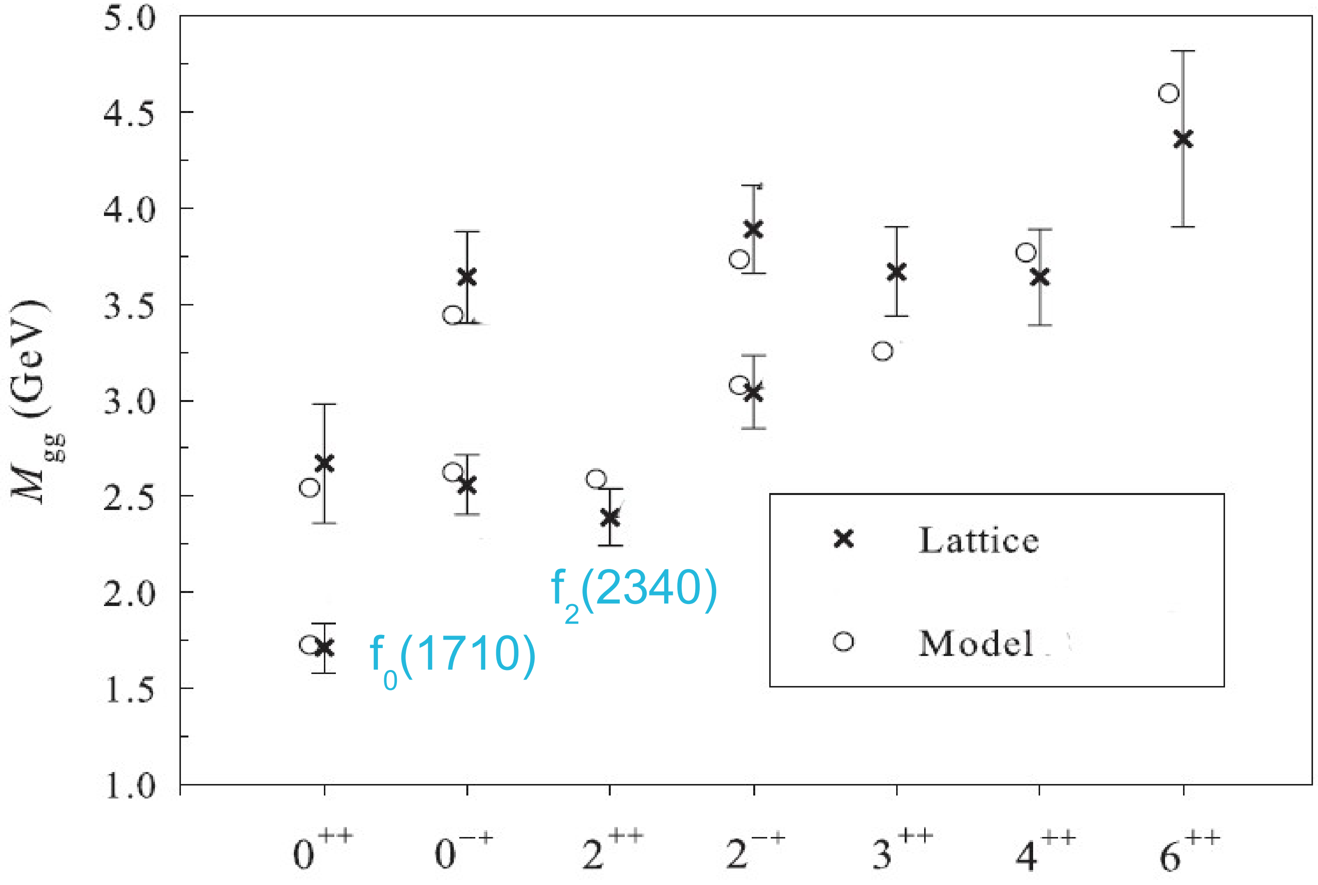


O. Philipsen and H. Wittig, Phys. Lett. B **451**, 146 (1999)

Glueball mass spectrum

Two gluons (I)

Mass spectrum



Two gluons (II)

Transverse gluons

- No light $J = 1$ state
- Expected number of states
- Good agreement
- Needed : relativistic kinematics

Longitudinal gluons

- Too many states
- Poor agreement

V. Mathieu, FB, and C. Semay, Phys. Rev. D **77**, 114022 (2008)

Three gluons (I)

Color

- $[[8,8]^{8_S}]^{1_S}$, C = –, symmetric spin-space
  - Lightest states
  - Like three photons
  - Transverse: No light (pseudo)scalar state

  F.G. Fumi, L. Wolfenstein, Phys. Rev. **90**, 498 (1953)

- $[[8,8]^{8_A}]^{1_A}$, C = +, antisymmetric spin-space

Problem: Wick's formalism

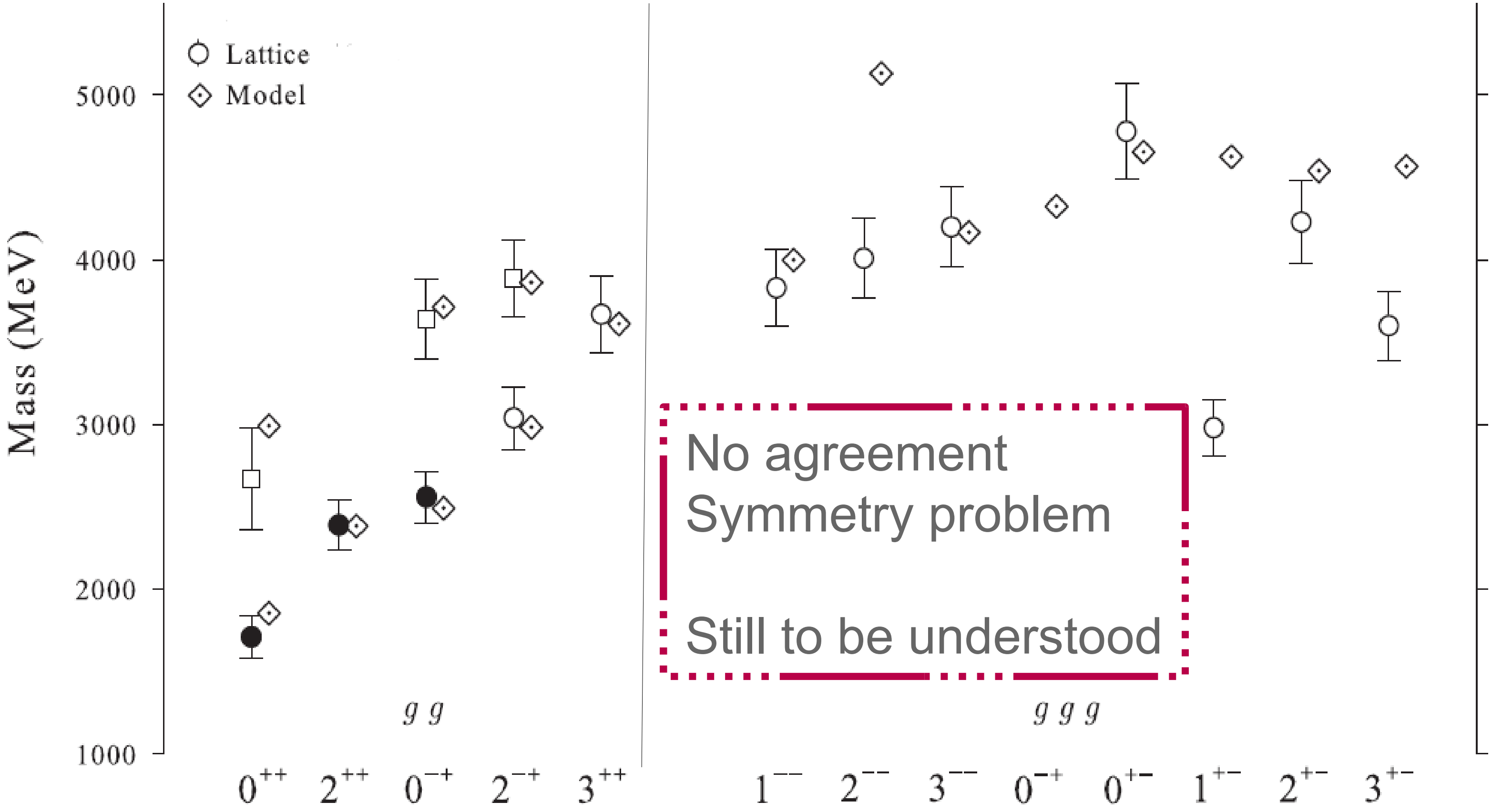
- Not available yet for three-gluon glueballs

G.C. Wick, Ann. Phys. (N.Y.) **18**, 65 (1962)

Three gluons (II)

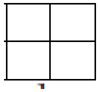
Mass spectrum with spin-1 gluons

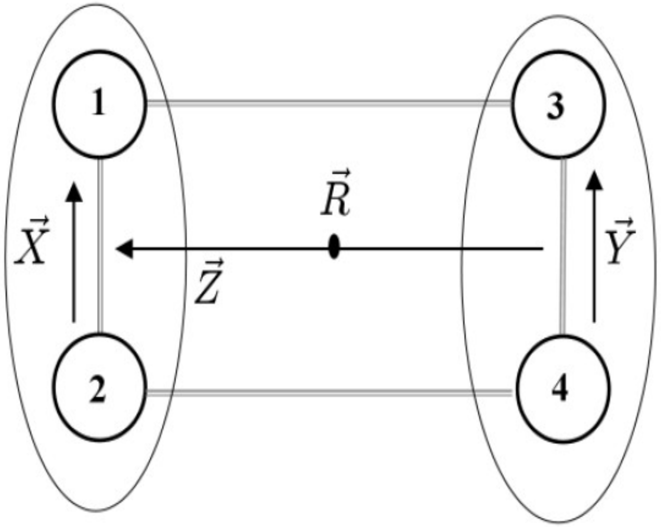
V. Mathieu, C. Semay, and B. Silvetsre-Brac, Phys. Rev. D **77,** 094009 (2008)



Four gluons?

A heavy 0^{+-} state seen on the lattice

- Highly excited three-gluon state
- Low-lying four-gluon state
  - Proposal, color function $[[8,8]^{10},[8,8]^{\overline{10}}]^1$
  - Symmetry 



Mass estimate, ok with lattice QCD

Many-boy helicity formalism needed

Large N limit

Strong coupling $\sigma = \frac{C_R}{N}\sigma_0$

L. Del Debbio, H. Panagopoulos, P. Rossi, and E. Vicari, JHEP**01**, 009 (2002)

- Invariant with N if R = Adjoint

One gluon exchange $\propto C_R \alpha_s \propto \frac{C_R}{N}\alpha_0$

- Invariant with N if R = Adjoint

⟼ Spectrum roughly invariant with N

- OK with recent lattice studies, up to SU(8)

B. Lucini, A. Rago, and E. Rinaldi, JHEP **1008**, 119 (2010)

Thermodynamics

Warming up

Increasing the temperature

$T = 0$

- « Usual QCD »
- Confinement
- Hadron gas

$T > T_c$

- QGP
- Deconfinement
- Quark – gluon gas

$T = T_c$

- Phase transition

Pure Yang-Mills similar to QCD

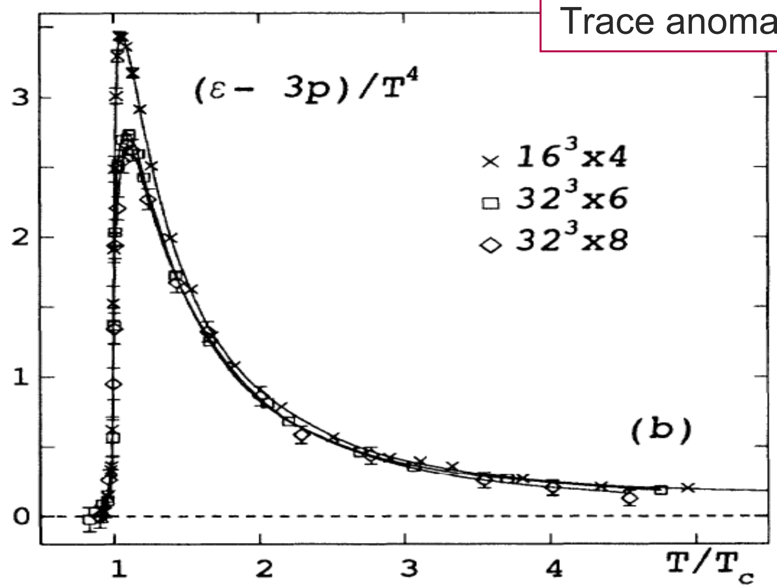
Equation of state

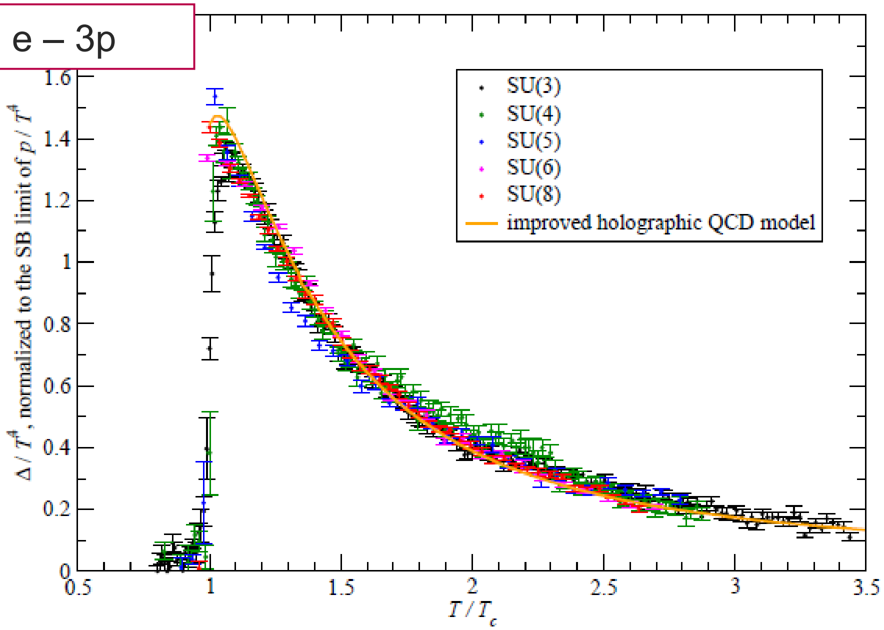
Results from the lattice

G. Boyd *et al.*, PRL **75**, 4169 (1995)

M. Panero, PRL **103**, 232001 (2009)

Trace anomaly ~ e – 3p





Phase transition, « weakly first order »

Quasiparticle models

Well above T_c

- Ideal gas of deconfined gluons
  - Thermal masses from perturbation theory
  - Scaling in $(N^2 - 1)$ as expected

Around T_c

- Strongly interacting gas of deconfined gluons
  - Maybe presence of glueballs
  - Not fully understood

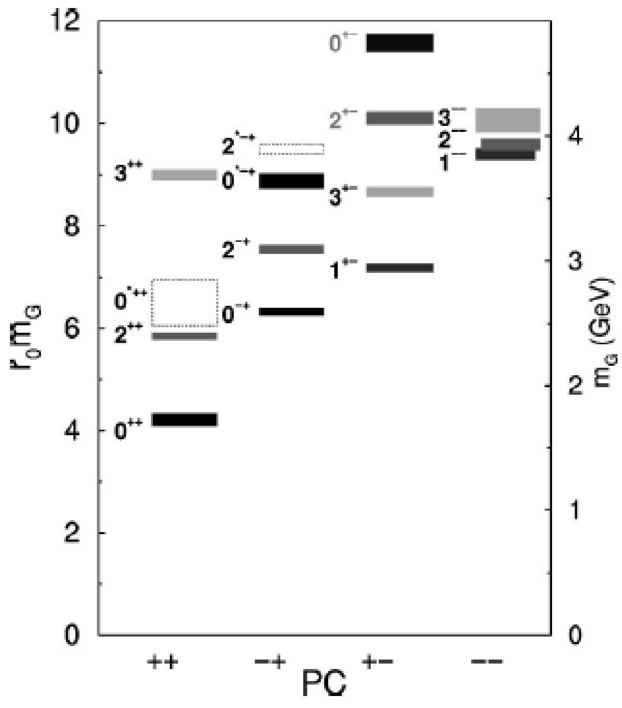
Below T_c

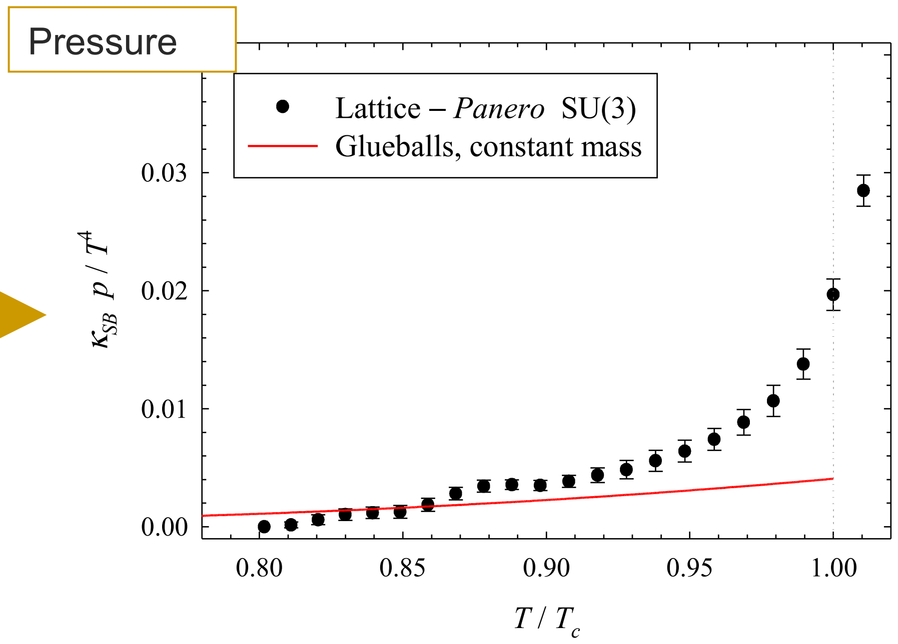
- Glueball gas
  - Not studied a lot

Simple glueball gas

Basic model: Ideal Bose gas

- Input, lattice spectrum + T_c = 300 MeV





Hagedorn spectrum (I)

Pressure underestimated

- Glueball pressure suppressed $\propto (2J+1)\,\mathrm{e}^{-m_G/T}$
- Negligible contribution of high-lying states

String picture of glueballs

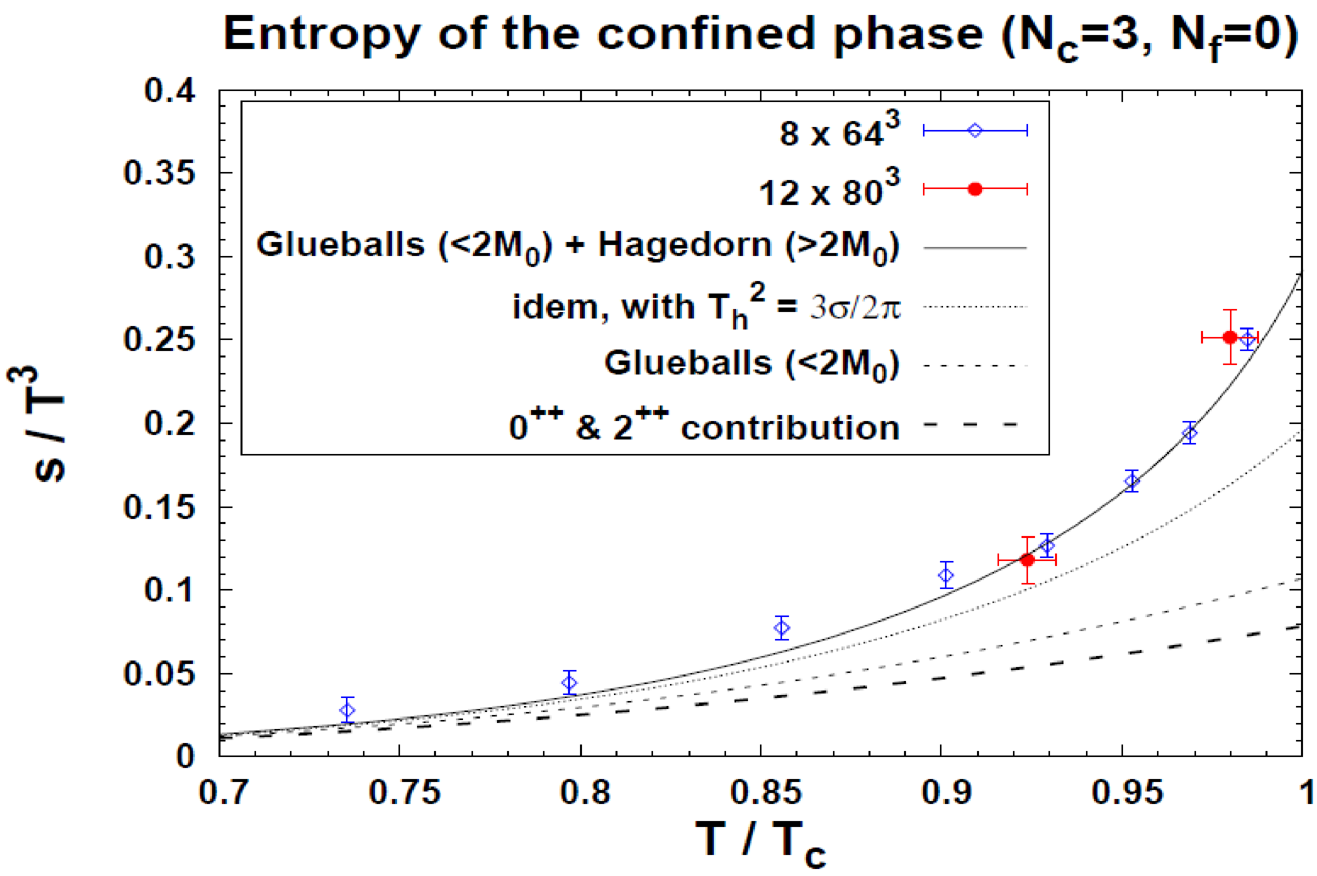
- String theory predicts a Hagedorn spectrum : Degeneracy growing like $\mathrm{e}^{+m_G/T}$
- Relevant contribution of high-lying states
- Might be suggested by experimental data (mesons and baryons)

  W. Broniowski, W. Florkowski and L. Y. Glozman, PRD **70**, 117503 (2004)

Hagedorn spectrum (II)

Agreement with lattice data

H. B. Meyer, PRD **80**, 051502(R) (2009)



$$\frac{T_h}{T_c} = 1.024$$

Conclusions

Summary (I)

- Glueballs : « QCD only »
  - Pure gauge bound states
- Lattice : various data available
  - Mass spectrum in different cases
  - Wave function
  - Thermodynamics
- Constituent models
  - Successfull for mesons and baryons
  - Mass spectrum partly agrees with lattice data
    - Standard Hamiltonian
    - Transverse gluons with relativistic kinematics
  - Glueball gas for gluonic matter below Tc

Outlook

Three-gluon bound states

- Need to deal with helicity states for three identical transverse bodies
- C = - sector in lattice still not understood

Experimental candidates

- Possibly seen in f_0 and f_2 resonances
- Probably not pure glue state
- Issue : « unquenching » the existing models
  - Much remains to be done

Very last slide

Lattice

- More fundamental
- Gives « all »
- Numerical

Constituent models

- Less fundamental
- Capture essential features
- Intuitive / analytical

Complete each other