Neutron stars as gravity laboratories

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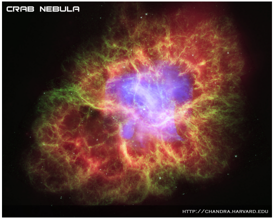
Modern Aspects of Gravity and Cosmology Workshop

Neutron stars are the results of stellar evolution.
We can see them in stellar remnants.
A typical example is the Crab nebula that hosts the Crab pulsar[a].

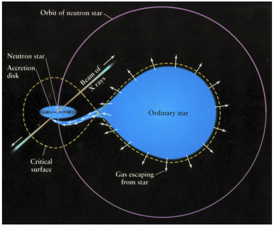
[a] APOD 2006 October 26



Very often we find rapidly rotating pulsars at the end of stellar evolution. The fastest rotating known pulsar (PSR J1748-2446ad) spins at 716Hz and it is part of a binary system[a].

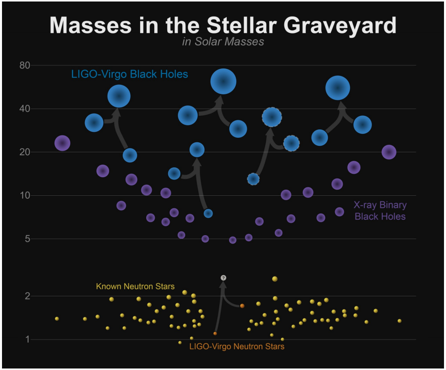
Low mass X-ray binaries are systems that are comprised by a compact object (NS or BH) and a regular star companion. The main source of the X-rays is the accretion disk that forms around the compact object.

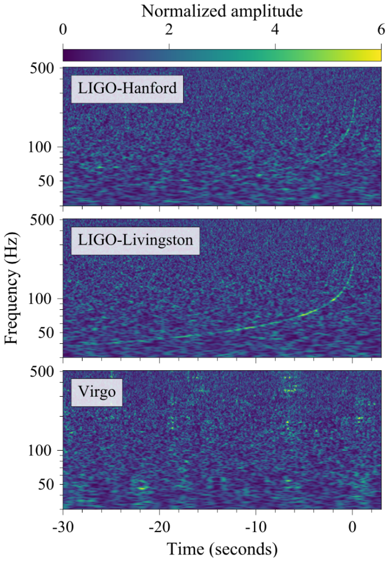
[a] J. W. T. Hessels et al., Science **311** 1901 (2006)



Interesting astrophysics takes place around NSs that depends on the background spacetime. Matter in their interior is at very high densities, where the equation of state is unknown. NSs have strong enough gravitational fields that can test our theories of gravity.

And as we all saw on Monday the 16th, 2017, from the 17th of August 2017 we have "heard", through gravitational waves, the inspiral and collision of a binary neutron star system.[a]

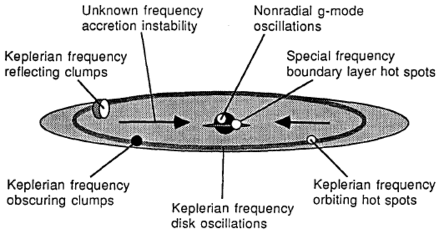




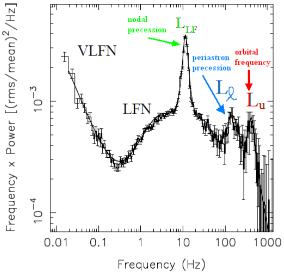
[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **119**, 161101 (2017)

In low-mass X-ray binaries we can have observables related to geodesic motion.
An example of observables related to orbits around neutron stars are the quasi-periodic oscillations (QPOs) of the spectrum[1] of an accretion disc.

Mechanisms for producing QPOs[2] from orbital motion



Typical X-Ray spectrum[3]



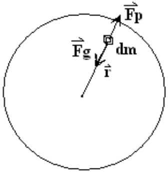
Effects: orbiting hot spots, oscillations on the disc, precessing rings or misaligned precessing discs, and so on. These could result in a modulated emission or they could be eclipsing the emission from the central object.

[1] Stella & Vietri, 1998, ApJ, 492, L59.

[2] F.K. Lamb, Advances in Space Research, 8 (1988) 421.

[3] Boutloukos et al., 2006, ApJ, 653, 1435-1444.

Neutron stars: Fluid configurations in equilibrium.



Newtonian Stars

Hydrostatic equilibrium (spherical symmetry):

$\nabla P = -\rho \nabla \Phi \Rightarrow \frac{dP}{dr} = -\frac{d\Phi}{dr}\rho = -G\frac{m(r)}{r^2}\rho$

Mass (spherical symmetry): $\frac{dm}{dr} = 4\pi \rho r^2$

Field equations: $\nabla^2 \Phi = 4\pi G \rho$,

Equation of state for the fluid: $P = P(\rho)$.

Relativistic non-rotating Stars

Instead of a gravitational field Φ, gravity is described by a metric g_{ab}.

In spherical symmetry:

$$ds^2 = -e^{2\Phi}dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Field equations: $G^{ab} = 8\pi G T^{ab}$,

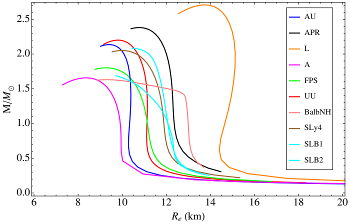
$\Rightarrow \Phi$: $\frac{d\Phi}{dr} = \frac{m(r) + 4\pi r^3 P}{r(r - 2m(r))}$,

Definition of the Mass: $\frac{dm}{dr} = 4\pi \rho r^2$,

Hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{\rho m(r)}{r^2}\left(1 + \frac{P}{\rho}\right)\left(1 + \frac{4\pi P r^3}{m(r)}\right)\left(1 - \frac{2m(r)}{r}\right)^{-1}$$

Equation of state for the fluid: $P = P(\rho)$.



Slowly Rotating Neutron Stars[a]

$$ds^2 = -e^{\bar{\nu}}\left(1+2\epsilon^2 h\right)dt^2 + e^{\lambda}\left[1+\frac{2\epsilon^2 m}{(r-2M)}\right]dr^2 + r^2\left[1+2\epsilon^2 k\right]\left[d\theta^2+\sin^2\theta(d\phi-\epsilon\omega dt)^2\right].$$

where $\epsilon = \Omega/\Omega^*$ is the slow rotation small parameter with respect to $\Omega^* = (M/R^3)^{1/2}$.

[a] Hartle J. B., Thorne K. S., ApJ **153**, 807 (1968)

Rapidly Rotating Neutron Stars: Numerical

The line element for a stationary and axially symmetric spacetime is[a],

$$ds^2 = -e^{2\nu}dt^2 + r^2\sin^2\theta B^2 e^{-2\nu}(d\phi-\omega dt)^2 + e^{2(\zeta-\nu)}(dr^2+r^2d\theta^2).$$

Komatsu, Eriguchi, and Hechisu[b] proposed a scheme for integrating the field equations which is implemented by the RNS numerical code to calculate rotating neutron stars. [c]

[a] E. M. Butterworth and J. R. Ipser, ApJ **204**, 200 (1976).

[b] H. Komatsu, Y. Eriguchi, and I. Hechisu, MNRAS **237**, 355 (1989).

[c] N. Stergioulas, J.L. Friedman, ApJ, **444**, 306 (1995).

Rapidly Rotating Neutron Stars: Analytic

Using the Weyl-Papapetrou line element that describes stationary and axisymmetric vacuum spacetimes,

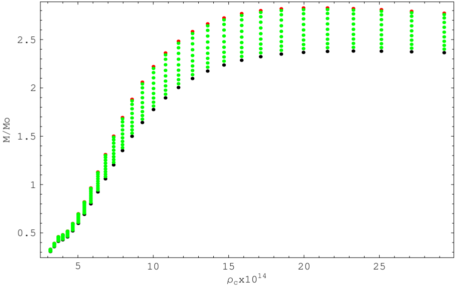
$$ds^2 = -f\left(dt-\omega d\phi\right)^2 + f^{-1}\left[e^{2\gamma}\left(d\rho^2+dz^2\right)+\rho^2 d\phi^2\right],$$

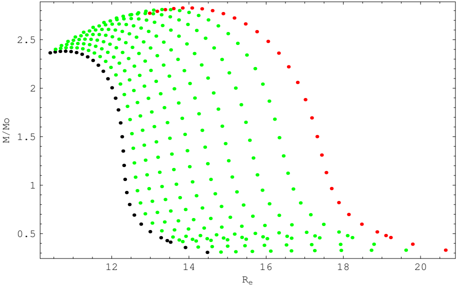
Ernst[a] reformulated the Einstein field equations to take the form, $(Re(\mathcal{E}))\nabla^2\mathcal{E} = \nabla\mathcal{E}\cdot\nabla\mathcal{E}$, using the complex potential $\mathcal{E}(\rho,z) = f(\rho,z)+i\psi(\rho,z)$, where $f=\xi^a\xi_a$ and ψ is defined by, $\nabla_a\psi = \varepsilon_{abcd}\,\xi^b\nabla^c\xi^d$.

[a] F.J. Ernst, Phys. Rev., **167**, 1175 (1968); Phys. Rev., **168**, 1415 (1968).

Results from numerical models:

One can use RNS to calculate models of rotating neutron stars for a given equation of state. For example we show here some models for the APR EoS:





The models with the fastest rotation have a spin parameter, $j = J/M^2$, around 0.7 and a ratio of the polar radius over the equatorial radius, r_p/r_e, around 0.56.

The code calculates the various physical characteristics of the NS, the metric functions on a grid, and the relativistic multipole moments, i.e., M, $S_1 \equiv J$, $M_2 \equiv Q$, $S_3 \equiv J_3$ and M_4. [a] These moments characterise the NS and the spacetime around it.

[a] G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Neutron star multipole moments in GR

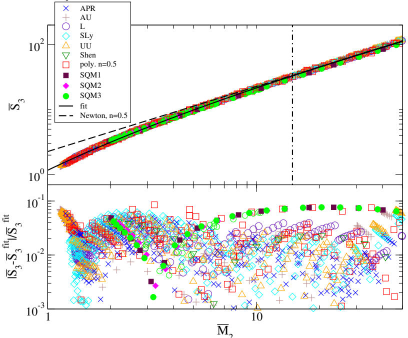
Black Hole-like behaviour of the moments:[4]

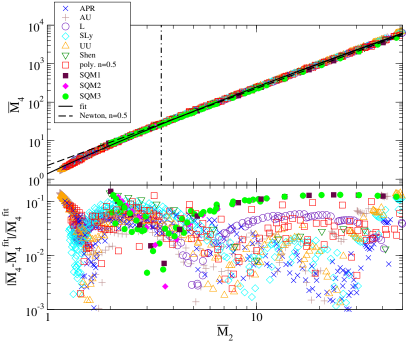
Kerr moments	Neutron star moments
$M_0 = M,$	$M_0 = M,$
$J_1 \equiv J = jM^2,$	$J_1 = jM^2,$
$M_2 \equiv Q = -j^2 M^3,$	$M_2 = -a(EoS, M)j^2 M^3,$
$J_3 \equiv S_3 = -j^3 M^4,$	$J_3 = -\beta(EoS, M)j^3 M^4,$
$M_4 = j^4 M^5,$	$M_4 = \gamma(EoS, M)j^4 M^5,$
$\vdots$	$\vdots$
$M_{2n} = (-1)^n j^{2n} M^{2n+1},$	$M_{2n} = ?,$
$J_{2n+1} \equiv S_{2n+1} = (-1)^n j^{2n+1} M^{2n+2}$	$J_{2n+1} = ?$

where $j = J/M^2$.

[4] W.G. Laarakkers and E. Poisson, Astrophys. J. **512** 282 (1999); G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

EoS independent behaviour of the moments[5] :



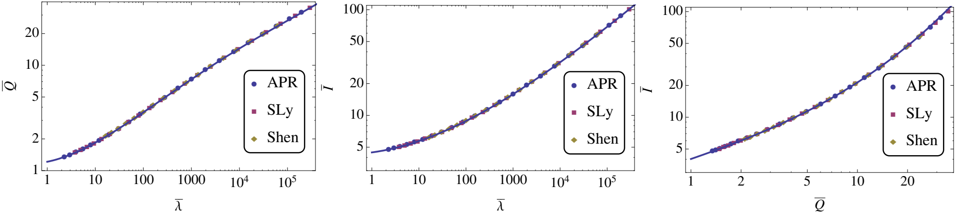


$$\bar{M}_{2n} = |M_{2n}/(j^{2n}M^{2n+1})|, \ \bar{S}_{2n+1} = |S_{2n+1}/(j^{2n+1}M^{2n+2})|$$

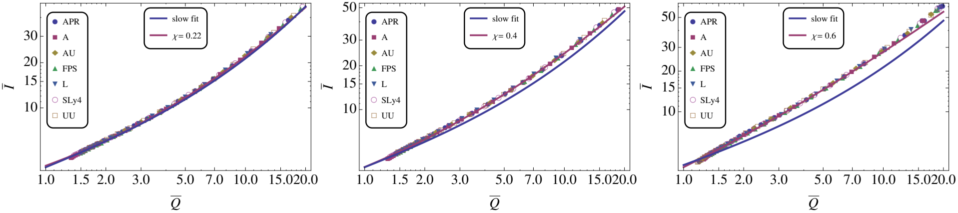
All these are properties that characterise the spacetime around neutron stars as well as the gravitational aspects of the stars themselves. Therefore we can use them to construct analytic neutron star spacetimes with only few parameters.

[5] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Slow rotation Q-Love, I-Love and $I - Q$ relations[6]



Slow and rapid rotation $I - Q$ relations ($\chi \equiv j$)[7]



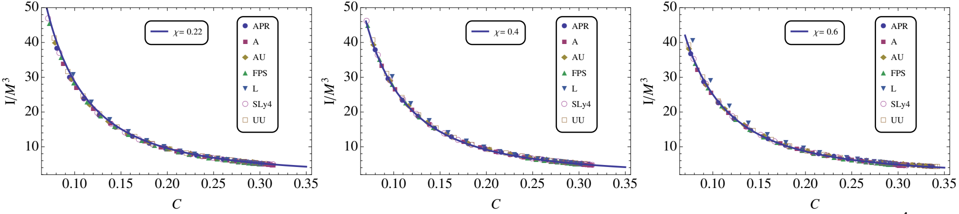
[6] K. Yagi and N. Yunes, Science 341, 365 (2013); Phys. Rev. D 88, 023009 (2013).

[7] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014)

$I - \mathcal{C}$ relations:

First studied by Lattimer et al. and inspired by analytic models such as the Tolman VII model ($\rho = \rho_c[1-(r/R)^2]$).

$I - \mathcal{C}$ relations for different rotation rates[8]



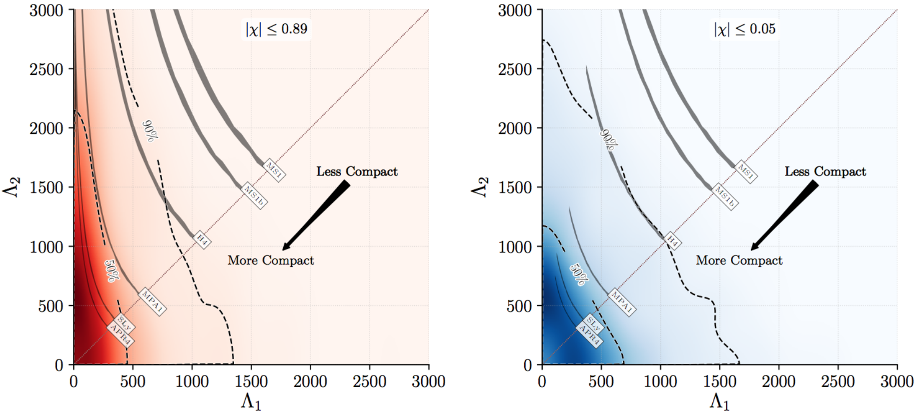
$$I/M^3 = (1.471+0.448\chi) - \frac{0.0802+0.27289\chi}{\mathcal{C}} + \frac{0.438-0.0346\chi}{\mathcal{C}^2} - \frac{0.01694+0.0056\chi}{\mathcal{C}^3} + \frac{(3.316+1.57\chi)\times 10^{-4}}{\mathcal{C}^4},$$

where $\mathcal{C} = M/R$ is the compactness.

[8] C. Breu and L. Rezzolla, MNRAS 459, 646 (2016); K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016); Daniela D. Doneva, GP, arXiv:1709.08046 [gr-qc].

From the gravitational wave phase one can extract information for the Love numbers of each of the two stars, Λ_1 and Λ_2. In the fig. stars with larger radii are towards up and right. This gives an estimate of $R \lesssim 14km$ for an $1.4M_\odot$ NS.[a]

[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **119**, 161101 (2017)



In the case of Scalar-Tensor theories with a massless scalar field,

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-2\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi\right)+S_m(g_{\mu\nu},\psi)\,,$$

the field equations in the Einstein frame take the form,

$$\tilde{R}_{ab}=2\partial_a\phi\partial_b\phi+8\pi G\left(T_{ab}-\frac{1}{2}g_{ab}T\right),\quad \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\phi=-4\pi\alpha(\phi)T$$

These equations can be solved as in GR in order to construct neutron stars.[9] On the other hand, the vacuum field equations can admit an Ernst formulation as in GR,[10]

$$(Re(\mathcal{E}))\nabla^2\mathcal{E}=\nabla\mathcal{E}\cdot\nabla\mathcal{E},$$

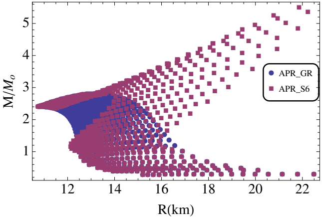
with the addition of a Laplace equation for the scalar field $\nabla^2\phi=0$.
One can extend the definition of multipole moments in this case as well where the moments (mass, spin, scalar) are defined in the Einstein frame but the actual physics is done in the Jordan (physical) frame, where the metric is given by the conformal transformation $g_{\mu\nu}=A^2(\phi)\tilde{g}_{\mu\nu}$.[11]

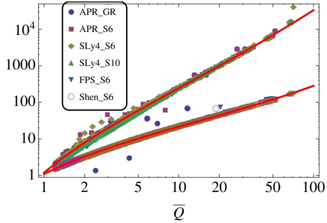
[9] D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Phys. Rev. D 88, 084060 (2013)

[10] GP, T.P. Sotiriou, Phys. Rev. D91, 044011 (2015)

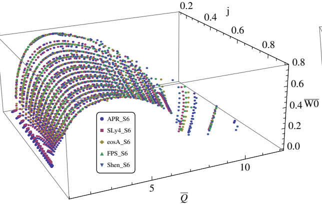
[11] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

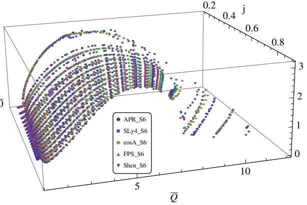
ST models against GR models for APR EoS and S_3^{ST}, M_4^{ST} vs Q^{ST} relations for various EoSs

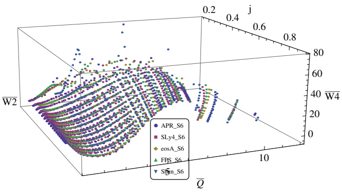




Scalar field normalised moments plotted against the spin parameter and the quadrupole,[12]







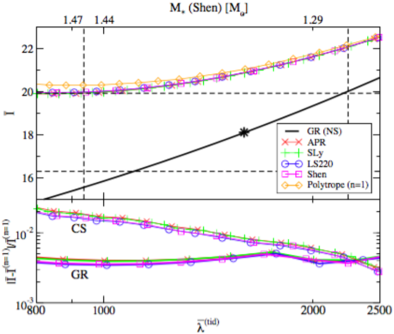
[12] D.D. Doneva, G.P., T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Work in progress.

I-Love-Q relations:

Such relations have been examined in dCS gravity, in massless and massive scalar-tensor theories (ST), in Einstein-dilaton-Gauss-Bonnet (EdGB) gravity, in Eddington-inspired Born-Infeld (EiBI) theory and in $f(R)$ theories.

$*$ The deviations from GR are almost negligible in the cases of the massless ST, the EdGB and the EiBI theory.

$*$ Larger differences on the other hand are observed for dCS gravity, massive ST and $f(R)$ theories.

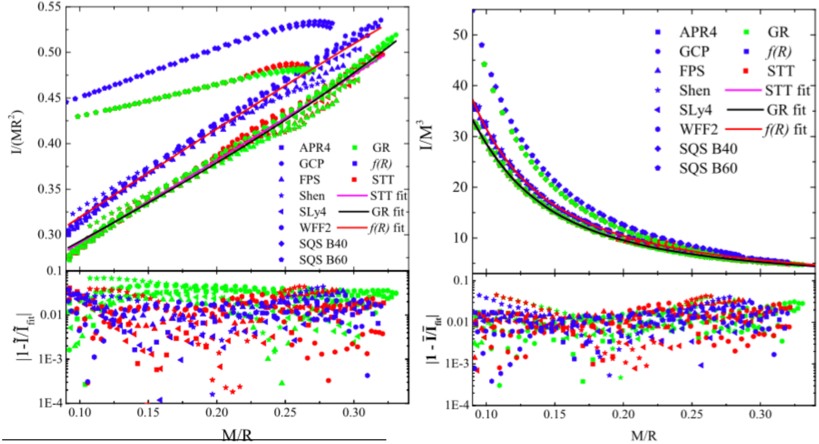


For example here we see how dCS compares to GR.[a]

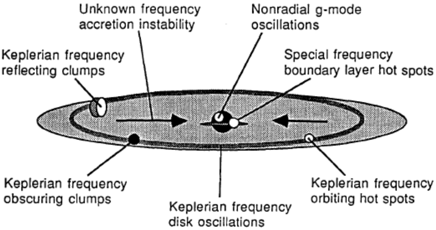
[a] K. Yagi and N. Yunes, Phys. Rev. D 88, 023009 (2013).

$I - \mathcal{C}$ relations:

Slow rotation models for a massless ST theory, an $f(R) = R + \alpha R^2$ theory and GR (in ST $\beta = -4.5$).[13]



[13] K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016).



Circular equatorial orbits: If we define $\Omega \equiv \frac{d\phi}{dt}$, the energy, angular momentum and orbital frequency for the circular orbits take the form,

$$\tilde{E} \equiv \frac{E}{m} = \frac{-g_{tt} - g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}},$$

$$\tilde{L} \equiv \frac{L}{m} = \frac{g_{t\phi} + g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}},$$

$$\Omega = \frac{-g_{t\phi,\rho} + \sqrt{(g_{t\phi,\rho})^2 - g_{tt,\rho}g_{\phi\phi,\rho}}}{g_{\phi\phi,\rho}}$$

For more general orbits: Equations of motion can take the general form,

$$-g_{\rho\rho}\left(\frac{d\rho}{d\tau}\right)^2 - g_{zz}\left(\frac{dz}{d\tau}\right)^2 = 1 - \frac{\tilde{E}^2 g_{\phi\phi} + 2\tilde{E}\tilde{L}g_{t\phi} + \tilde{L}^2 g_{tt}}{(g_{t\phi})^2 - g_{tt}g_{\phi\phi}} = V_{eff}.$$

We can study the precession properties from the properties of the effective potential.

$$-g_{\rho\rho}\left(\frac{d(\delta\rho)}{d\tau}\right)^2 - g_{zz}\left(\frac{d(\delta z)}{d\tau}\right)^2 = \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}(\delta\rho)^2 + \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial z^2}(\delta z)^2,$$

This equation describes two harmonic oscillators with epicyclic frequencies, $\bar{\kappa}_\rho^2 = \frac{g^{\rho\rho}}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}\Big|_c$, $\bar{\kappa}_z^2 = \frac{g^{zz}}{2}\frac{\partial^2 V_{eff}}{\partial z^2}\Big|_c$. The differences of these frequencies (corrected for redshift) from the orbital frequency, $\Omega_a = \Omega - \kappa_a$, define the precession frequencies.

The energy change per logarithmic frequency interval and the precession frequencies are related to the spacetime multipole moments (Ryan, 1995),

in GR:

$$\Delta\tilde{E} = -\frac{U}{3}\frac{d\tilde{E}}{dU} = \frac{1}{3}U^2 - \frac{1}{2}U^4 + \frac{20J_1}{9M^2}U^5 + \dots$$

$$\frac{\Omega_\rho}{\Omega} = 3U^2 - 4\frac{J_1}{M^2}U^3 + \left(\frac{9}{2} - \frac{3M_2}{2M^3}\right)U^4 - 10\frac{J_1}{M^2}U^5 + \left(\frac{27}{2} - 2\frac{J_1^2}{M^4} - \frac{21M_2}{2M^3}\right)U^6 + \dots$$

$$\frac{\Omega_z}{\Omega} = 2\frac{J_1}{M^2}U^3 + \frac{3M_2}{2M^3}U^4 + \left(7\frac{J_1^2}{M^4} + 3\frac{M_2}{M^3}\right)U^6 + \left(11\frac{J_1M_2}{M^5} - 6\frac{S_3}{M^4}\right)U^7 + \dots$$

where $U = (M\Omega)^{1/3}$.

The Orbital frequency gives the Keplerian mass: $\Omega = (M/r^3)^{1/2}(1 + O(r^{-1/2}))$.

in Scalar-Tensor theory:[a]

$$\Delta\tilde{E} = \frac{1}{3}U^2 + \left(\frac{2\beta_0 W_0^2}{9\bar{M}^2} - \frac{8\alpha_0 W_0}{9\bar{M}} - \frac{1}{2}\right)U^4 + \frac{20J_1}{9\bar{M}^2}U^5 + \dots$$

$$\frac{\Omega_\rho}{\Omega} = \left(3 - \frac{W_0\left(\beta_0 W_0 - 8\alpha_0\bar{M}\right)}{2\bar{M}^2}\right)U^2 - \frac{4J_1}{\bar{M}^2}U^3 + \dots$$

$$\frac{\Omega_z}{\Omega} = \frac{2J_1}{\bar{M}^2}U^3 + \frac{3(M_2 - \alpha_0 W_2)}{2\bar{M}^3}U^4 - \frac{2J_1W_0\left(\beta_0 W_0 - \alpha_0\bar{M}\right)}{\bar{M}^4}U^5 + \dots$$

where $U = (\bar{M}\Omega)^{1/3}$. The calculations are done in the Jordan frame. Again the orbital frequency gives the Keplerian mass: $\Omega = (\bar{M}/r^3)^{1/2}(1 + O(r^{-1/2}))$, but this time the Keplerian mass is $\bar{M} = M - W_0\alpha_0$. W_0 is the scalar charge, W_2 is the scalar quadrupole and $\alpha \equiv (d\ln A)/d\phi$, $\beta \equiv d\alpha/d\phi$. These observables could in principle distinguish between GR and Scalar-Tensor theory.

[a] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

- Neutron stars exhibit some black hole-like behaviour with respect to their moments structure, but the moments are different from black hole moments, so the geometry is essentially different from the geometry of Kerr black holes.
- There are several NS properties that show universal behaviour (EoS independent).
- These universal properties are present in several alternative theories of gravity as well, such as in ST theories.
- Some of these relations could be used to distinguish between some alternative theories.
- The multipole moments determine the orbital dynamics and are of relevance to the study of accretion discs and quasi periodic oscillations (QPOs).
- These geodesic properties could also distinguish between different theories of gravity such as GR and Scalar-Tensor theory.
- There is a lot of work to be done (in particular on the astrophysics side).

Thank you