Leptogenesis with Right-handed Neutrinos Compact Spectrum

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Summary

1) Motivations for Gauge Unification
and for Leptogenesis

2) Neutrino Oscillations and the Seesaw Model

3) Implications of a Compact N_R Spectrum
for Tritium Decay and $(\beta\beta)_{0\nu}$ effective mass

4) Numerical Analysis to realize Leptogenesis
with a Compact N_R Spectrum

5) Conclusion and Prospects

Gauge Unification

Why we believe that Gauge Unification is not Optional

Quantum Numbers of each Family : stronger motivation than believed for Gauge Unification beyond the Standard Model

$SU(3)\times SU(2)\times U(1)$	TrY	TrY^2	TrC_2	TrC_3
(3, 2, +1/6)	+1	1/6	9/2	8
($\bar{3}$, 1, -2/3)	-2	4/3	0	4
(1, 1, +1)	+1	1	0	0
Sum of traces	0	5/2	9/2	12
(1, 2, -1/2)	-1	1/2	3/2	0
($\bar{3}$, 1, +1/3)	+1	1/3	0	4
Sum of traces	0	5/6	3/2	4

TrY vanishes on the two sets (*Y* is a generator), but also sums in last three columns are in same ratio 3 $\longrightarrow$ two representations of same algebra

SU(5) and SO(10) Unified Gauge Theories

At the unification scale one would predict for the gauge couplings

$$\frac{\alpha}{\alpha_s} = \sin\theta_W^2 = \frac{3}{8}$$

- both relations in disagreement with experiment
- modified in the right direction by the renormalization group equations
- in agreement with the scale dependence of $\alpha_s(Q^2)$

Very elegant minimal $SU(5)$ model (Georgi-Glashow) disproved by

- proton lifetime (looking at $p \to e^+ \ \pi^0$)
- meeting points of $g_i(Q^2)$ (i = 3, 2, 1) of $SU(3) \times SU(2) \times U(1)$ are different : first (g_1, g_2) meet, and at higher energies (g_2, g_3) meet (at a one scale consistent with proton decay)

SUSY version of $SU(5)$ model

- modifies these predictions in the right direction
- approximate meeting of three gauge couplings at high scale

- atractive feature of SUSY for a solution of "hierarchy problem"
- this decreased interest for modifications of $SU(5)$ minimal model

From $SU(5)$ to $SO(10)$ Grand Unification

Most appealing is the extension of gauge group $SU(5)$ to $SO(10)$

- implies existence of L antineutrinos (by CPT of R neutrinos !)
- each family in spinorial representation 16
- decomposition under $SU(5)$: $16 \rightarrow 10 + \bar{5} + 1$
- intermediate Pati-Salam symmetry $SU(4) \times SU(2)_L \times SU(2)_R$

$SO(10)$ **Grand Unification**

• $SU(5)$ predictions modified in agreement with experiment at the price of spontaneous $B - L$ breaking at $\sim 10^{11}$ GeV

• two elegant features of SO(10) :

- opposite anomalies of 10 and $\bar{5}$ $SU(5)$ representations are consequence of the fact that $SO(10)$ has no anomaly (the cubic Casimir $T_{ab}T_{bc}T_{ca}$ vanishes !)

- $SO(10)$ contains $SU(5)$, obvious generalization of $SU(3) \times SU(2) \times U(1)$, but also SU(4) x $SU(2)_L$ x $SU(2)_R$

- quarks and leptons $\in$ (4, 2, 1) and their antiparticles $\in$ ($\bar{4}$, 1, 2)

• the most appealing feature of $SO(10)$ is its natural explanation of the tiny values of the masses of left-handed neutrinos

Fermion masses in SU(5)

• Stressed success of $SU(5)$ with electroweak Higgs $\in 5 + \bar{5}$:
lepton and down quark mass matrices coincide at unification scale
this implies for the third family $m_b = m_\tau$
good prediction since $\frac{m_b}{m_\tau}$ decreases with scale : $\frac{m_b}{m_\tau} \simeq 2$ at M_Z

• Prediction gets better in SUSY $SU(5)$

or

by requiring equality at the scale of $SU(4) \times SU(2) \times SU(2)$
symmetry breaking $\sim 10^{11} GeV < 10^{15} GeV$

• Assumption that EW Higgs $\in 5 + \bar{5}$ representations of $SU(5)$
provides right quark-colour content of fundamental fermions
that follows by demanding that $SU(3)_c \times U(1)_Q \subset SU(4)$
which is the little group of all vectors of the representation 5

The Seesaw Model

$SO(10)$ with the corresponding assumption that
the electroweak Higgs transforms as $10 \rightarrow 5 + \bar{5}$
leads to equality of Dirac neutrino and up quark mass matrices
giving the disastrous predictions

$$m_t = m_{\nu_\tau} \qquad m_c = m_{\nu_\mu} \qquad m_u = m_{\nu_e}$$

Natural in $SO(10)$ the **seesaw model** relates the Majorana mass matrix of L neutrinos m_L to the Dirac neutrino mass matrix m_D and the Majorana mass matrix of R neutrinos M_R

$$m_L = -m_D \frac{1}{M_R} m_D^t$$

this formula implies for L neutrinos masses which are many orders of magnitude smaller than those of charged fundamental fermions

Baryogenesis and Leptogenesis

• No evidence in the explored universe of macroscopic presence of antimatter : it would produce dramatic annihilation processes

• A positive value of the baryon number $Y_B = n_B - n_{\bar{B}} > 0$ is required in the framework of the presently accepted inflationary cosmological model

• Primordial nucleosynthesis depends on the relative abundance of baryons with respect to photons, which dissociate deuteron nuclei and disfavour the formation of helium nuclei

• Relative primordial abundance of H, He, Li and Be implies

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6 \times 10^{-10}$$

• In agreement with cosmic microwave background anisotropies

Leptogenesis and Neutrino Oscillations

• Violation of baryon number implied by gauge unified theory turned the nightmare of proton stability into the attractive feature of being a necessary condition to produce baryon asymmetry

• In minimal $SU(5)$ one is not able to produce a sufficient Y_B and at the electroweak scale non-perturbative contributions, **sphalerons**, which violate $B + L$, but not $B - L$, wash out any B asymmetry produced at a higher scale with $B - L = 0$, as **in $SU(5)$ where $B - L$ is a global symmetry**

• Instead $B - L$ **is a generator of $SO(10)$** that must be **spontaneously broken** at an intermediate scale

• In particular one may consider a scenario called **Leptogenesis** where a leptonic asymmetry in the out-of-equilibrium decay of the lightest right-handed neutrino is turned into asymmetries in both B and L by sphalerons at the electroweak scale

Leptogenesis and Neutrino Oscillations

• From the role of right-handed neutrinos in leptogenesis and in the seesaw model, and data on solar and atmospheric neutrinos, Falcone and Tramontano (2001) concluded that the MSW solution with large mixing is the one for which it is easier to realize the leptogenesis scenario (before evidence in favour of that solution)

• Akhmedov, Frigerio and Smirnov (2003), assuming a hierarchical spectrum for Dirac neutrino masses, got a sufficient lepton asymmetry from an approximate degeneracy of the two lightest right-handed neutrinos (the heaviest ν_R has a mass $\sim 10^{14}$ GeV)

• We look for a leptogenesis scenario with a compact spectrum and ν_R masses $\sim 10^{11}$ GeV, the scale of spontaneous symmetry breaking of $B-L$ in $SO(10)$, and also the scale of ν_R masses in the SUSY $SO(10)$ model with breaking induced by $54+45+16$ Higgs into $SU(3)\times SU(2)\times U(1)$: $\quad M_{\nu_R} \sim \frac{|<16>|^2}{M_{Planck}}$

Consequences of the Demand of a Compact ν_R Spectrum

We take as eigenvalues of the Dirac neutrino mass matrix the values assumed by AFS (Akhmedov et al., 2003), reasonably inspired by the masses of quarks with charge 2/3

$m_{D_1} = 1\ MeV \qquad m_{D_2} = 400\ MeV \qquad m_{D_3} = 100\ GeV$

From the seesaw formula

$$Det(M_R) = -\frac{Det(m_D)^2}{Det(m_L)}$$

and the cosmological upper limit

$$\sum_i m_{\nu_i} < 0.2\ eV$$

found by the Bari group (Fogli et al., 2006)
one gets the lower limit

$$|Det(M_R)| > (1.7 \times 10^9\ GeV)^3$$

A Lower Limit for the Ratio of the Masses of the two Heavier Left-handed Neutrinos

Since in principle there is no lower limit on the mass of the lightest left-handed neutrino there is no upper limit for Det (M_R)

A reasonable guess may be obtained from data on solar and atmospheric neutrino oscillations

$$\Delta m_s^2 \simeq 8 \times 10^{-5}\ eV^2 \qquad \Delta m_a^2 \simeq 2.5 \times 10^{-3}\ eV^2$$

This implies, **within the normal hierarchy**, the lower limit for the ratio of the two heaviest left-handed neutrinos

$$|\frac{m_2}{m_3}| \geq 0.18$$

that means the reasonable guess $\quad Det(m_L) \simeq (\Delta m_s^2)^{1.5}$

giving $\quad Det(M_R) = (1.3 \times 10^{10}\ GeV)^3$

near the value expected for $B - L$ breaking at $\sim 10^{11}\ GeV$

The Mass Scale for Right-handed Neutrinos

• The hierarchical spectrum of Dirac masses should imply a still more hierarchical spectrum both for L and for R neutrinos

• This property does not hold for the left-handed neutrinos as shows the lower limit $|\frac{m_2}{m_3}| \geq 0.18$

• A compact right-handed neutrino spectrum means also to avoid a hierarchical spectrum for ν_R

• The Dirac neutrino mass matrix $m_D = V^{L+} m_D^{diag} V^R$ implies the **inverse seesaw** formula

$$M_R = -V^{Rt} m_D^{diag} V^{L*} m_L^{-1} V^{L+} m_D^{diag} V^R$$

or $$M_R = -V^{Rt} m_D^{diag} A^L m_D^{diag} V^R$$

the matrix $$A^L = V^{L*} m_L^{-1} V^{L+}$$

plays an important practical role in our calculations

Phase counting

- In general, the Seesaw model has 6 independent CP phases
- Assuming $s_{13} \simeq 0$, the light neutrino mass is diagonalized by

$$U \simeq \begin{pmatrix} c_s & s_s & 0 \\ -\frac{s_s}{\sqrt{2}} & \frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_s}{\sqrt{2}} & -\frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \, \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$$

- In $SO(10)$ one has the relation $\qquad V^R = V^{L*}$

Because of approximate quark-lepton symmetry and diagonal mass matrix for charged leptons, V^L can be parametrized by

$$V^L = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_L} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_L} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_L} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_L} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_L} & c_{23}c_{13} \end{pmatrix}$$

We have three independent phases : δ_L and the Majorana α, β

Tuning Conditions for a Compact ν_R Spectrum

To have a compact spectrum for right-handed neutrinos one needs very small contributions to M_R of order $(m_{D_3})^2$ and $m_{D_2} m_{D_3}$

To avoid these contributions one needs $|A^L_{33}|$ and $|A^L_{23}|$ very small

Let us take the very drastic conditions $\quad A^L_{33} = A^L_{23} = 0$

With the values $\quad \tan\theta_a = 1 \qquad \tan^2\theta_s = .4$

and the simple Cabibbo form $\quad V^L = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

one finds

$$A^L_{33} = -\frac{1}{2}\left(\frac{2}{7m_1} + \frac{5}{7m_2} + \frac{1}{m_3}\right)$$

$$A^L_{23} = -\frac{\cos\theta}{2}\left(\frac{2}{7m_1} + \frac{5}{7m_2} - \frac{1}{m_3}\right) + \frac{\sqrt{5}\sin\theta}{7}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)$$

therefore :

$$\frac{m_2}{m_1} = -\frac{1+\sqrt{0.2}\tan\theta}{0.4-\sqrt{0.2}\tan\theta} \qquad \frac{m_3}{m_2} = 1 - \frac{\sqrt{0.8}}{\tan\theta}$$

The Consequences for m_L

With $m_1 = .003$ eV and $\sin\theta_{12} = .14$ we find reasonable values

$\Delta m_s^2 = 8.1 \times 10^{-5}\ eV^2$ $\qquad$ $\Delta m_a^2 = 2.5 \times 10^{-3}\ eV^2$

The scale of neutrino masses and the Majorana phases are fixed with predicted values well below the present upper bounds

One has for the sum of the neutrino masses,
the kinematical mass appearing in tritium decay
and the effective mass appearing in neutrinoless double beta decay

Model	Upper experimental limit
$\Sigma m_{\nu_i} = .063$ eV	$< 0.2\ eV$
$m_{\nu_e} = .005$ eV	$< 2.2\ eV$
$m_{ee} = .0006$ eV	$< 0.4\ eV$

The Leptonic Asymmetry

To get leptonic asymmetry able to realize the Leptogenesis scenario

- We take the more general V^L (of the order of the CKM matrix)
- Need to relax the homogeneous equations in $1/m_i$ $(i = 1, 2, 3)$

$$A^L_{33}(\{1/m_i\}) = A^L_{23}(\{1/m_i\}) = 0$$

allowing for small inhomogeneous terms

$$A^L_{33}(\{1/m_i\}) = C^L_{33} \qquad\qquad A^L_{23}(\{1/m_i\}) = C^L_{23}$$

with $|C^L_{33}|$, $|C^L_{23}|$ sufficiently small to keep the property of a compact spectrum for right-handed neutrinos

Each term in $A^L_{33}(\{1/m_i\}), A^L_{23}(\{1/m_i\})$ is of $O(10^{11}\ GeV^{-1})$

Therefore we need $\qquad |C^L_{33}|, |C^L_{23}| << 10^{11}\ GeV^{-1}$

If C^L_{33} and C^L_{23} are supposed real : only one independent CP phase

Heavy and Light neutrino Spectra

Finally we find solutions with

• masses for the ν_R around $10^{10} GeV$

• left-handed neutrinos with a mass matrix similar to the one found in the approximation with only $s_1 \neq 0$

• Majorana phases slightly different from the values ± 1

In conclusion, we get pratically the same predictions (well below the present upper limits)

for the three quantities Σm_{ν_i}, m_{ν_e} and m_{ee}

associated to the mass matrix of left-handed neutrinos

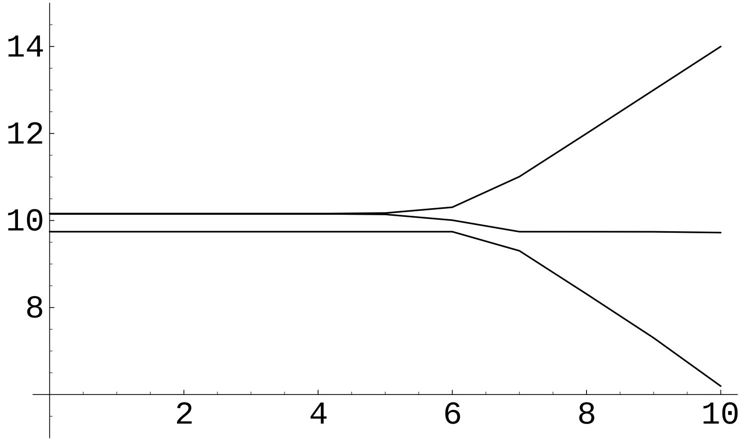

Fig. 1. Log-log plot of the right-handed heavy neutrino spectrum (in GeV) as a function of $-C^L_{23} = C^L_{33} > 0$ (in GeV^{-1}) for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$.
Between $-C^L_{23} = C^L_{33} = 10^6 - 10^7\ GeV^{-1}$ there is a level crossing.
Angular points come from interpolation of a finite number of points.

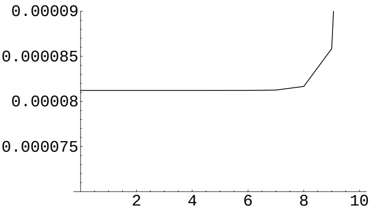

Fig. 2

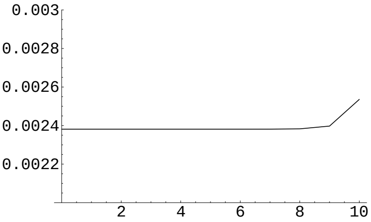

Fig. 3

Fig. 2. Δm_s^2 (in eV^2) as function of $-C_{23}^L = C_{33}^L > 0$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$, in a log scale for C_{33}^L.

Fig. 3. Δm_a^2 (in eV^2) as function of $-C_{23}^L = C_{33}^L > 0$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$, in a log scale for C_{33}^L.

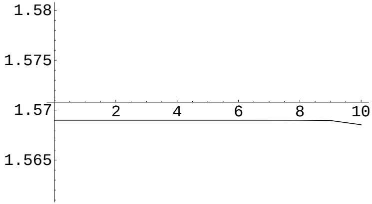

Fig. 4

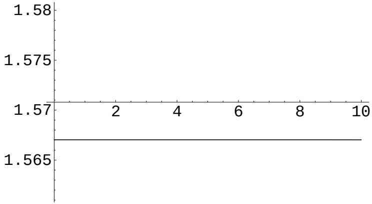

Fig. 5

Fig. 4. Majorana phase α as function of $-C^L_{23} = C^L_{33} > 0$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$, in a log scale for C^L_{33}. The x-axis is centered at $\frac{\pi}{2}$.

Fig. 5. Majorana phase β as function of $-C^L_{23} = C^L_{33} > 0$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$, in a log scale for C^L_{33}. The x-axis is centered at $\frac{\pi}{2}$.

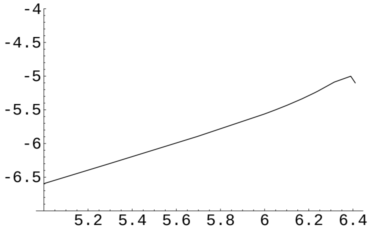

Fig. 6

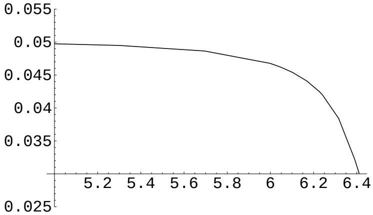

Fig. 7

Fig. 6. Log-log plot of the CP violation $-\epsilon_1$ as function of $-C^L_{23} = C^L_{33}$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$.

Fig. 7. Effective neutrino mass $\tilde{m}_1$ (in eV), controlling the washout, as function of $-C^L_{23} = C^L_{33}$ (in GeV^{-1}), in a log scale for C^L_{33}, for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$.

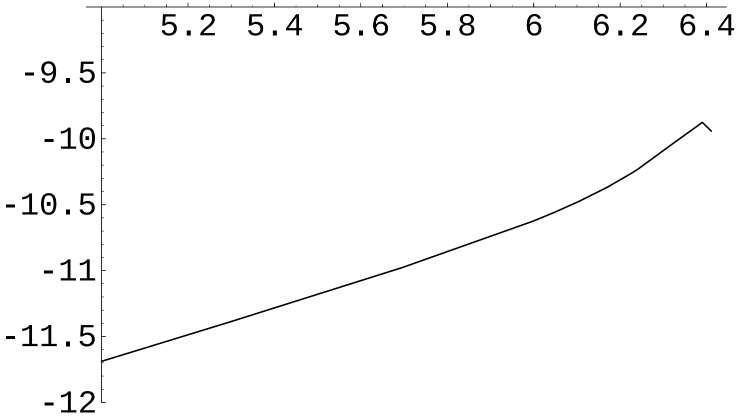

Fig. 8. Log-log plot of Y_{B_1} as a function of $-C^L_{23} = C^L_{33}$ (in GeV^{-1}), for $m_1 = 0.0030\ eV$ and $\tan\theta_{12} = 0.140$.

Conclusions

• The demand of a ν_R compact spectrum at a scale $\sim 10^{11}$ GeV motivated by SO(10) can produce at this scale the lepton asymmetry needed for leptogenesis in the seesaw scenario

• Consequences on ν_L observables $\Sigma m_{\nu_i} = .063$ eV $(< 0.2$ $eV)$, $m_{\nu_e} = .005$ eV $(< 2.2$ $eV)$, $m_{ee} = .0006$ eV $(< 0.4$ $eV)$

• The value for Σm_{ν_i} is close to the bound of cosmological observations : a factor two improvement of present upper bound may exclude almost degenerate neutrino masses

• Evidence for the three quantities one order of magnitude larger than predicted would be inconsistent with the proposed scenario

• Next steps :
- consider non vanishing θ_{13} for ν_L mixing (another CP phase)
- study of flavour effects for this compact ν_R spectrum