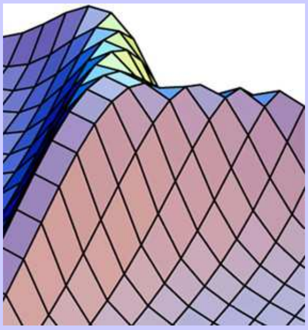
Towards the continuum limit of the lattice Landau-gauge gluon and ghost propagators

I.L. Bogolubsky (JINR, Dubna),
E.–M. Ilgenfritz (HU Berlin),
M. Müller-Preussker (HU Berlin),
A.Sternbeck (Regensburg University)



Funded by DFG - RFFI
FOR 465, 436 RUS 113/866

ICHEP 2010, Paris, July 2010

Abstract: We present recent results for the Landau gauge gluon and ghost propagators both in $SU(2)$and $SU(3)$ pure gauge theory for lattice sizes up to 112^4 corresponding to physical volumes up to $(15.8 \text{ fm})^4$. Considerable attention is paid to finite-volume, finite-size and Gribov copy effects. We employ a gauge-fixing method that combines a simulated annealing algorithm with finalizing overrelaxation. In the infrared region $q^2 \leq 0.01 \text{ Gev}^2$ we find the gluon propagator to become flat as a function of q^2. The ghost dressing function seems to tend to a constant value in the deep infrared, while running coupling α_s goes to zero for $q^2 \to 0$. In $SU(2)$ case we study transition to continuum limit using sequence of lattices with growing L keeping physical volume fixed.

Introduction

- Nonperturbative studies of Landau gauge gluon and ghost propagators

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}\,, \quad G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}$$

with continuum Dyson-Schwinger (DS) or Funct. Renorm. Group (FRG) Eqs. and within the lattice approach hopefully will provide consistent results.

- DS and FRG Eqs. have conformal solution
  [von Smekal, Hauck, Alkofer '98; Zwanziger '02; Lerche, von Smekal '02]

$$J(q^2) \propto (q^2)^{\alpha_{gh}} \quad \text{and} \quad Z(q^2) \propto (q^2)^{\alpha_{gl}}$$

with $$\alpha_{gl} + 2\alpha_{gh} = 0,$$

$D(q^2) = \frac{Zq^2}{q^2} \to 0, \quad J(q^2) \to \infty$ for $q^2 \to 0$.

It is argued to be unique, when DS combined with FRG Eqs. [Fischer, Pawlowski, '07]

- DS Eqs. provide also a decoupling solution
  [Boucaud et al. '05 - '07; Aguilar et al. '04 - '08]

- We present some further steps towards IR limit for the Landau gauge gluon and ghost propagators in quenched QCD on very large lattices.

SU(3)

Gauge fixing: standard approach

In order to fix the Landau gauge we apply a gauge transformation $g(x)$ to link variables $U_{x,\mu} \in SU(3)$ such that the gauge functional is maximized

$$F_U[g] = \sum_{x,\mu} \frac{1}{3} \,\mathfrak{Re}\,\mathrm{Tr}\; {}^g U_{x,\mu} \,.$$

$\Rightarrow$ For $A_\mu(x+\hat{\mu}/2) := (1/2ig_0)\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)_{\mathrm{traceless}}$
this is equivalent to $\Delta_\mu A_\mu = 0\,,$

$\Rightarrow$ but not unique: Gribov copies,

$\Rightarrow$ search for global maxima -
fundamental modular region (FMR).

Standard prescription:

i) $g(x)$ taken with periodic b.c.'s,

ii) maximize $F_U[g]$ with overrelaxation (OR) method.

Drawbacks of OR:

i) substantial slowing down of OR convergence with increasing lattice extension L,

ii) its possibilities to find global maximum of $F_U[g]$ are strongly limited.

Simulated annealing: the principle

- Simulated annealing (SA) is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ – allowing quasi-equilibrium tunnelings through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, "tied" to the (initially chosen) local maximum).

- Control parameters at hand:

  i) N_{iter}, T_{max} and T_{min},

  ii) schedule for temperature steps $T_i,\ i = 1, ..., N_{iter}$ can be optimized.

$\Rightarrow$ The larger N_{iter} the higher the local maxima, $N_{iter} \to \infty \quad \Longrightarrow$ global maximum.

$\Rightarrow$ Schedule in practice: $T_{max} = 0.45, T_{min} = 0.01$, $N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with tiny (larger) T-steps close to T_{max} (close to T_{min}).

Lattice Faddeev-Popov operator

Lattice Faddeev-Popov operator can be written in terms of the (gauge-fixed) link variables $U_{x,\mu}$ as

$$M^{ab}_{xy} = \sum_{\mu} A^{ab}_{x,\mu}\,\delta_{x,y} - B^{ab}_{x,\mu}\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}\,\delta_{x-\hat{\mu},y}$$

with

$$\begin{aligned} A^{ab}_{x,\mu} &= \quad \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^b T^a\,U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^a T^b\,U_{x-\hat{\mu},\mu}\right] \end{aligned}$$

and T^a, $a = 1,\ldots,8$ being the (hermitian) generators of the $\mathfrak{su}(3)$ Lie algebra satisfying $\mathrm{Tr}\left[T^a T^b\right] = \delta^{ab}/2$.

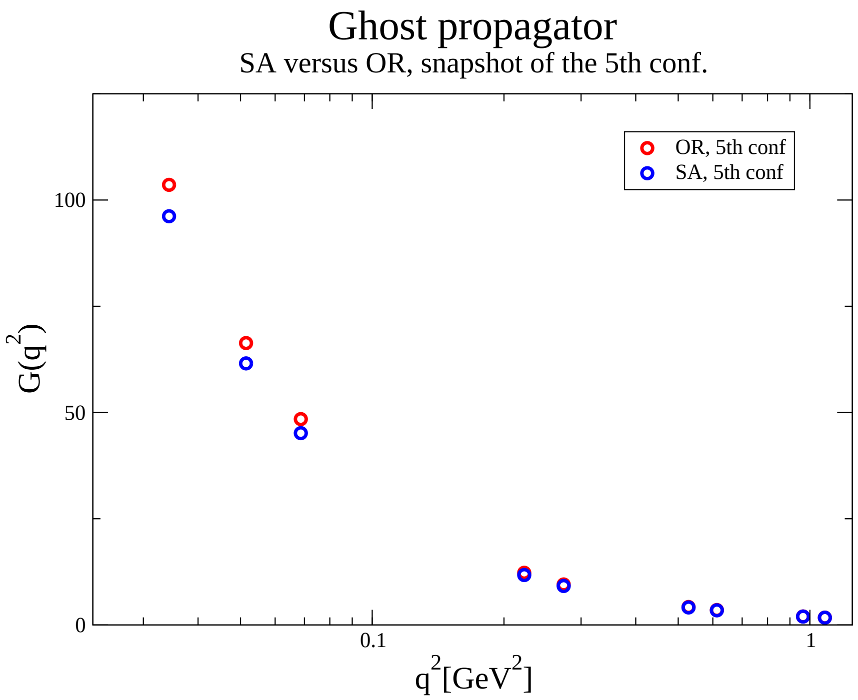
The ghost propagator is given by

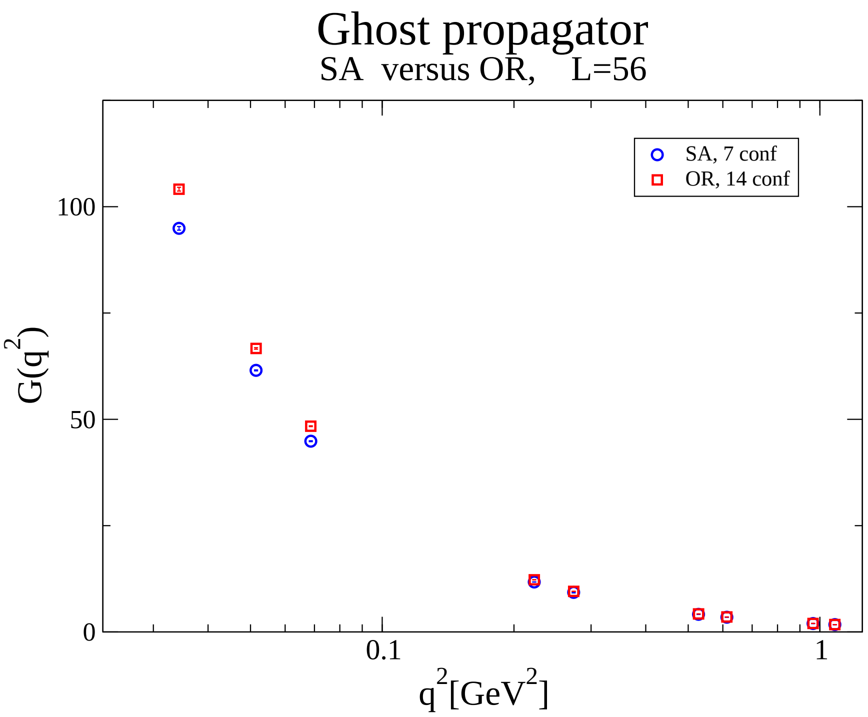
$$G^{ab} = \sum_{x,y} \left\langle \mathrm{e}^{-ik\cdot(x-y)} [M^{-1}]^{ab}_{x,y} \right\rangle$$

M-inversion with conjugate gradient method and plane wave sources.

Gauge fixing: SA vs. OR

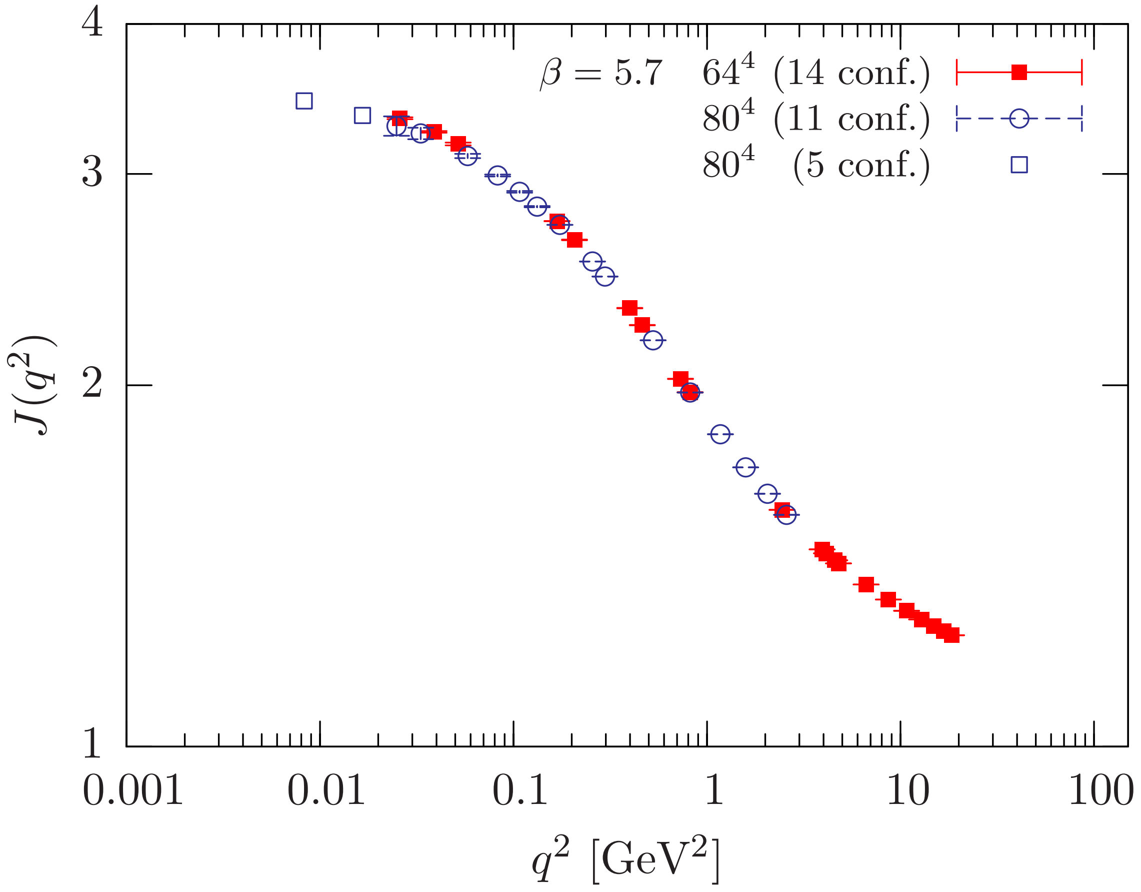
$SU(3)$ ghost propagator for $\beta = 5.70, \quad L = 56$





$\Rightarrow$ Influence of Gribov copies clearly visible, but seems to be moderate

Ghost : SA results, $\beta = 5.70$



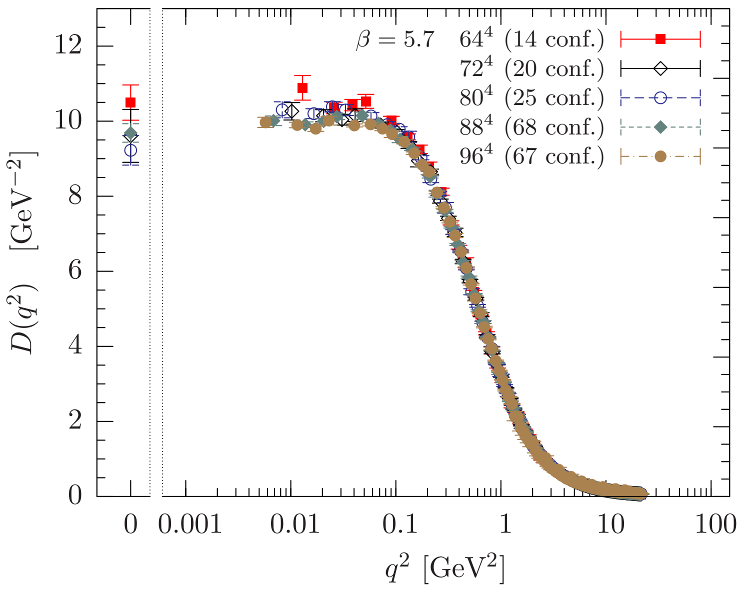
$\Rightarrow$ Weak estimator's dependence on MC configuration.

$\Rightarrow$ Finite-size effects of ghost propagator are very small and do not agree with finite-volume DS results [Fischer, Pawlowski '07].

$\Rightarrow$ No power-like asymptotics visible, i.e. differs from DS conformal solution with $\alpha_{gh} \approx 0.595$.

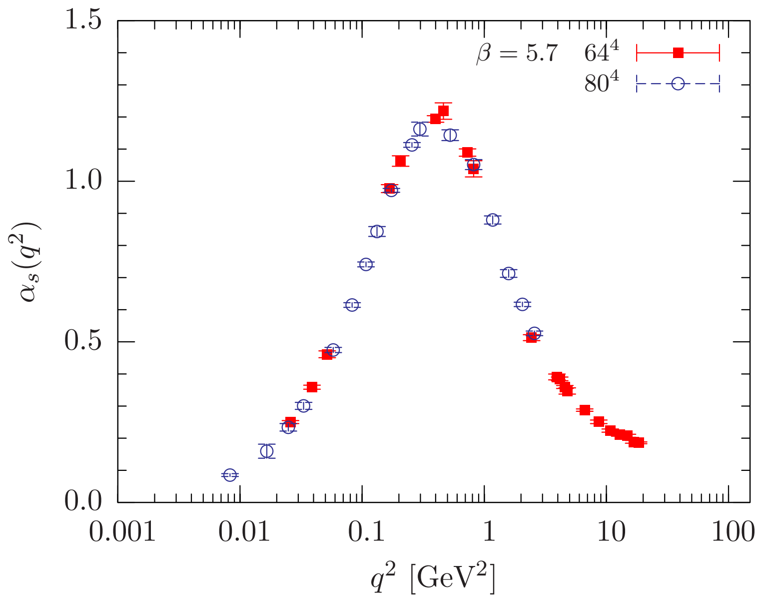
$\Rightarrow$ Lattice evidence for IR-regular ghost dressing function in agreement with the regular, *decoupling* DS solutions

Gluon propagator and running coupling



$\Rightarrow$ Flattening is clearly seen.

$\Rightarrow$ Results seem to support plateau hypothesis with $\alpha_{gl} = 1$ and $\alpha_{gl} + 2\alpha_{gh} \neq 0$.



$\Rightarrow$ No IR fixed point seen for running coupling

$$\alpha_s(q^2) \; = \; \frac{g_0^2}{4\pi} \; J^2(q^2) \; Z(q^2).$$

Conclusions and Questions

- Our lattice results seem to support the decoupling DS solution for the Landau gauge gluon and ghost propagators and to contradict the conformal one.

- Gribov copy effects seem to be moderate, but are still visible for the ghost propagator.
  Open question: Influence of enlargement of gauge orbits (e.g. with $Z(N)$ flips) and its influence on the finite-size behaviour.

- Weaknesses of the lattice approach:
  – in the IR the continuum limit not under control,
  – BRST invariance not properly treated
  – choice of the potential A_μ not unique,
  – choice of the boundary conditions not unique
  (here always periodic).

- Rôle of zero-momentum modes? Can they be suppressed by proper choice of A_{mu} and/or boundary conditions with non-periodic gauge transformations as shown in the $U(1)$ case ?
  [Bogolubsky et al., '00]

%===================================

SU(2)

SU(2): suppressing artefacts

Sources of distortions are:

⇒ . Finite-volume effects

⇒ Finite-size effects

⇒ Gribov copy effects.

⇒ Zero-momentum modes.

Simulated annealing: the principle

- Simulated annealing (SA) is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ – allowing quasi-equilibrium tunnelings through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, "tied" to the (initially chosen) local maximum).

- Control parameters at hand:

  i) N_{iter}, T_{max} and T_{min},

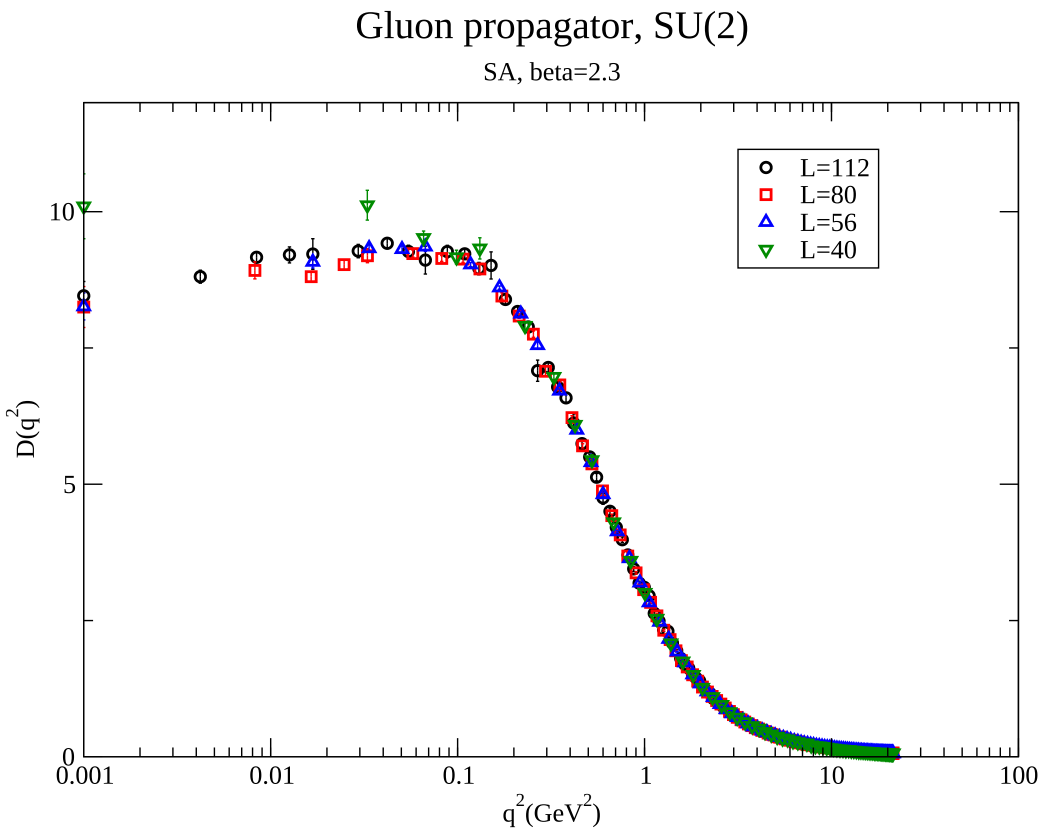
  ii) schedule for temperature steps $T_i,\ i = 1, ..., N_{iter}$ can be optimized.

$\Rightarrow$ The larger N_{iter} the higher the local maxima,
$N_{iter} \to \infty \quad \Longrightarrow \quad$ global maximum.

$\Rightarrow$ Schedule in practice: $T_{max} = 1.1$ for $SU(2)$, $T_{min} = 0.01$,
$N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with smaller (larger) T-steps close to T_{max} (close to T_{min}).

$SU(2)$ gluon, nonrenormalised

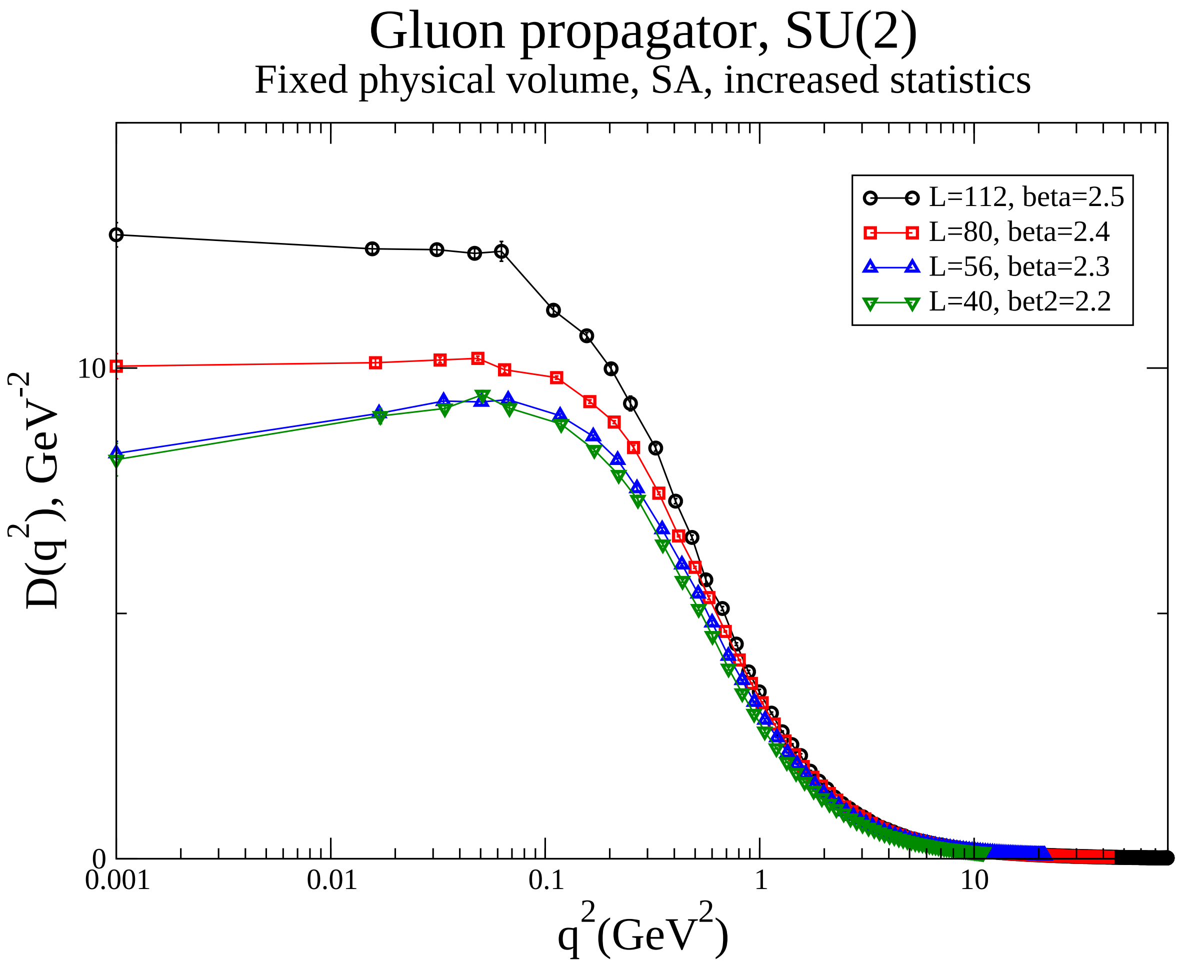
$SU(2)$ gluon propagator for $L = 40, 56, 80, 112$, $\beta = 2.3$



$\Rightarrow$ Finite-volume effects are small for L.ge.56

$SU(2)$ Gluon propagators

$SU(2)$ nonrenormalized gluon propagator for fixed physical volume, various $L = 40, 56, 80, 112$

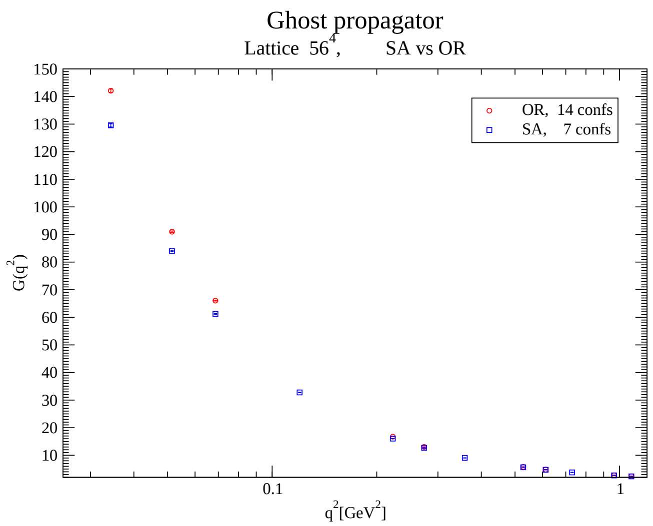


$\Rightarrow$ Finite-size effects are small for L.ge.56 and beta.ge.2.4

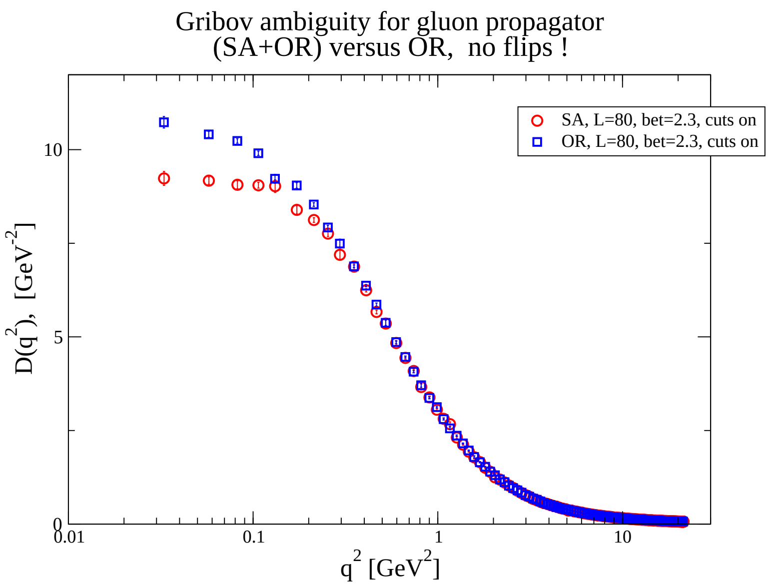
$\Rightarrow$ We are close to continuum limit!

Gauge fixing: SA vs OR

$SU(3)$ ghost propagator for $\quad \beta = 5.70, \quad L = 56$

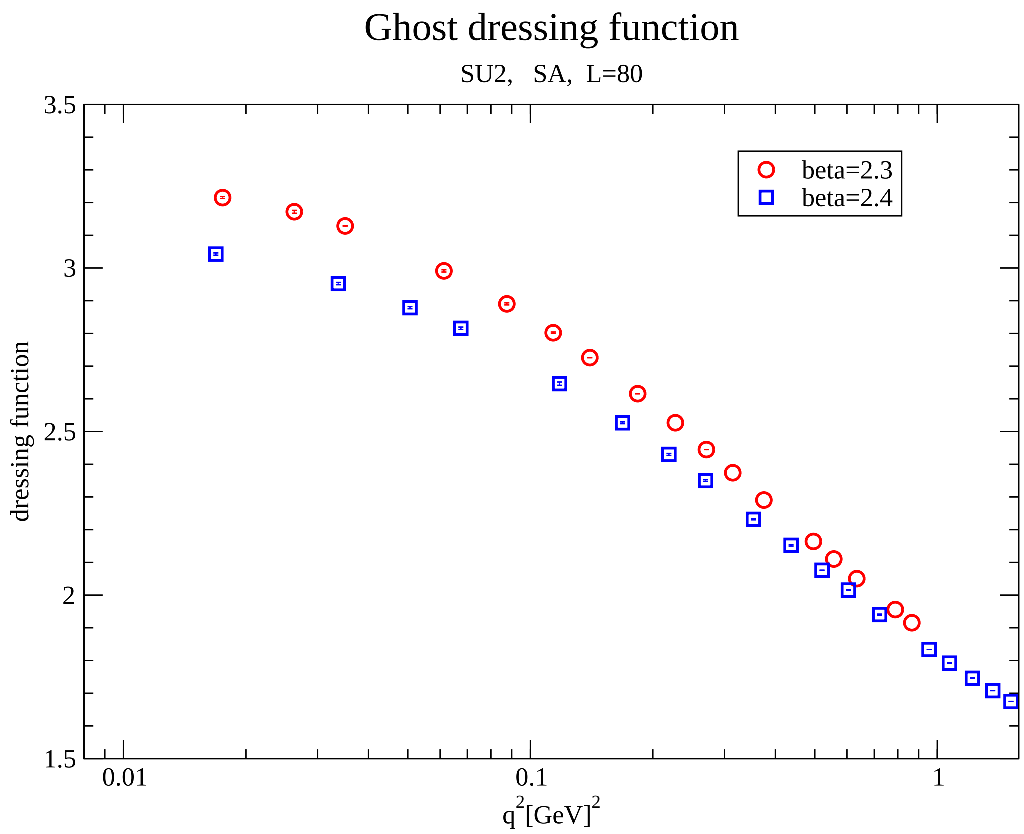


$SU(2)$ gluon propagator for $\quad \beta = 2.3, \quad L = 80$



$\Rightarrow$ Influence of Gribov copies clearly visible,
NEW! : for gluon the effect is already seen when comparing SA vs OR, without applying flip gauge transformation !

SU(2) Ghost, L=80



⇒ . Finite-size effects for beta=2.4 are small!

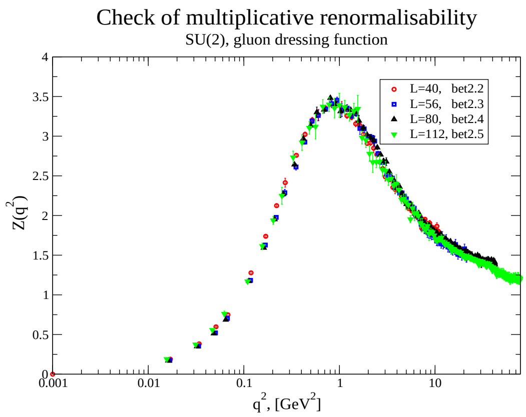
⇒ Require renormalization !.

⇒ Again no power-like asymptotics visible, i.e. differs from DS scaling solution.

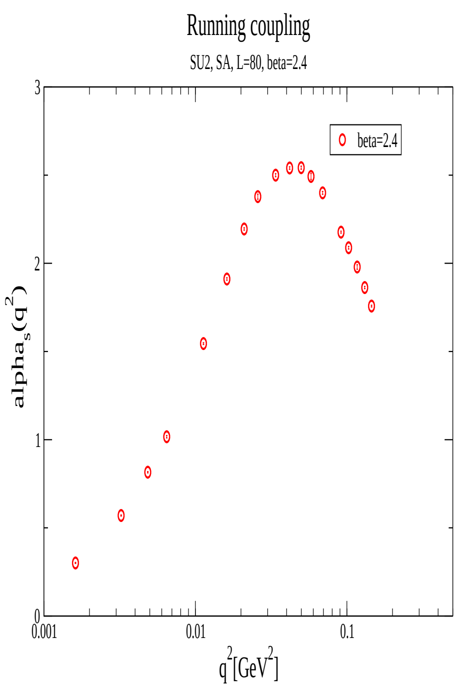
⇒ Again clearly seen plateau of ghost dressing function in agreement with the decoupling DS solutions

Gluon dressing function and $\alpha_s(q^2)$

We check for gluon in $SU(2)$ "muliplicative renormalisation" see [Bloch et al,'03]



$\Rightarrow$ Multiplicative renormalisation seems to hold !



$\Rightarrow$ No IR fixed point seen for $SU(2)$ running coupling

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} J^2(q^2)\, Z(q^2).$$

References

1. L. von Smekal, R. Alkofer, A. Hauck, PRL 79 (1997) 3591.
2. L. von Smekal, A. Hauck, R. Alkofer, Ann. Phys. 267 (1998) 1.
3. C. Lerche, L. von Smekal, PRD 65 (2002) 125006.
4. D. Zwanziger, PRD 65 (2002) 094039; PRD 69 (2004) 016002.
5. C.S. Fischer, A. Maas, J.M. Pawlowski, L. von Smekal, Annals Phys. 322 (2007) 2916.
6. C.S. Fischer, J.M. Pawlowski, PRD 75 (2007) 025012.
7. P. Boucaud et al. (2005), hep-ph/0507104.
8. Ph. Boucaud, J.P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, J. Rodriguez-Quintero, JHEP 0806 (2008) 012; 099.
9. A.C. Aguilar, A.A. Natale, JHEP 0408 (2004) 057.
10. A.C. Aguilar, D. Binosi, J. Papavassiliou (2008), arXiv:0802.1870 [hep-ph].
11. I.L. Bogolubsky, V.K. Mitrjushkin, M. Müller-Preussker, P. Peter, N.V. Zverev, PLB 476 (2000) 448.
12. T.D. Bakeev, E.-M. Ilgenfritz, V.K. Mitrjushkin, M. Müller-Preussker, PRD 69 (2004) 074507.
13. A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, A. Schiller, PRD 72 (2005) 014507.
14. A. Sternbeck, Ph.D.Thesis (2006), hep-lat/0609016.
15. A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, A. Schiller, I.L. Bogolubsky, PoS LAT2006 (2006) 076.
16. E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, A. Schiller, I.L. Bogolubsky, Brazilian Journal of Physics 37, N1B (2007) 193.
17. I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, PoS LAT2007 (2007) 290.
18. I.L. Bogolubsky, V.G. Bornyakov, G. Burgio, E.-M. Ilgenfritz, M. Müller-Preussker, V.K. Mitrjushkin, PRD 77 (2008) 014504, Erratum-ibid. D77 (2008) 039902.
19. See also A. Maas, A. Cucchieri, T. Mendes, Braz. J. Phys. 37N1B (2007) 219.
20. A. Cucchieri, T. Mendes, PoS LAT2007 (2007) 297; PRL 100 (2008) 241601.
21. I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker,A. Sternbeck, arXiv:0901.0736 [hep-lat].